Analytical models for assessing the influence of soil-structure interaction (SSI) on the natural frequencies of OWTs

Workshop on dynamics of wind turbines - Universidade de Brasília

Associate Professor Guilherme Rosa Franzini
Offshore Mechanics Laboratory
Departament of Structural and Geotechnical Engineering
Escola Politécnica, University of São Paulo, Brazil

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Offshore
Mechanics
Laboratory

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Offshore
Mechanics
Laboratory

Objectives

- Analytical models for assessing the influence of the soil-structure interaction (SSI) on the natural frequencies of OWTs;
- To present results from these models, with comparison with FEM and experimental results

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Dra. Ynaê Almeida (Assist. Prof. at UFBA)

- Ynaê concluded her PhD thesis under my supervision (March, 2023);
- Strong collaboration from Guilherme Vernizzi and Prof. Marcos Massao Futai (former supervisor);
- 2018-2020: PhD. thesis entitled "Modelos analíticos para cálculo de frequências naturais de torres eólicas considerando interação solo-estrutura.";
- Results from the thesis: Ferreira, Y.A.; Vernizzi, G.J.; Futai, M.M.; Franzini, G.R. A reduced-order model to predict the natural frequencies of offshore wind turbines considering soil-structure interaction, *Marine Systems & Ocean Technology*, v. 17, 80-94, 2022. The paper can be downloaded from here.

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Offshore Mechanics Laboratory

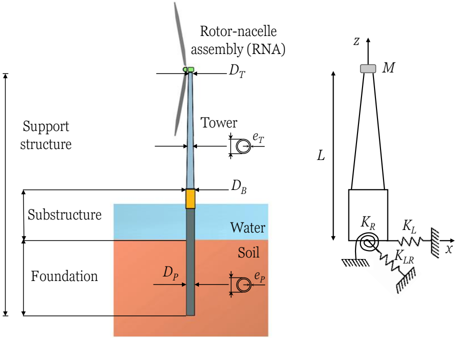
Hypotheses

- Soil representation: Stiffness matrix at the tip of the substructure;
- The forces due to SSI: Only conservative loads are considered;
- Tower: Tappered beam;
- Planar motion, all the materials have linear-elastic rheologic behavior, the RNA is considered as a mass at the tip of the tower;
- Small rotations $\Rightarrow$ Linear equations of motion are obtained.

Nomenclature

- A, I: cross-section area and moment of inertia;
- M: RNA mass;
- ρ: Material specific mass;
- m_1, m_2: Linear mass for the substructure and the tower, respectively.
- u, w: transverse and lateral displacements;

Problem description



Extracted from Ferreira *et al.* (2022)

- Equations of motion obtained using Hamilton's principle:

$$\int_{t_1}^{t_2} (\delta\mathcal{K} - \delta\mathcal{U} + \delta W^{nc})\mathrm{d}t = 0 \qquad (1)$$

- For the problem at hand: $\delta W^{nc} = 0$.

Formulation

- The kinetic energy $\mathcal{K}$:

$$\mathcal{K} = \frac{1}{2}M(\dot{u}^2_{(L)} + \dot{w}^2_{(L)}) + \frac{1}{2}\int_0^L \rho A(\dot{u}^2 + \dot{w}^2)dz \tag{2}$$

- After some manipulations and using the classical condition of the Variational Calculus:

$$\int_{t_1}^{t_2} \delta\mathcal{K}dt = -\int_{t_1}^{t_2} \left(M\left(\ddot{u}_{(L)}\delta u_{(L)} + \ddot{w}_{(L)}\delta w_{(L)}\right) + \int_0^L \rho A\left(\ddot{u}\delta u + \ddot{w}\delta w\right)\mathrm{d}z\right)\mathrm{d}t \tag{3}$$

- First variation of the gravitational energy

$$\delta\mathcal{U}^g = Mg\delta w_{(L)} + \int_0^L \rho A g \delta w dz \tag{4}$$

Formulation

- Terms associated with strain energy $\delta\mathcal{U}^s$ are:

$$\mathcal{U}^s = \iiint_V \frac{1}{2}\sigma\varepsilon dV = \iiint_V \frac{1}{2}E\varepsilon^2 dV \quad (5)$$

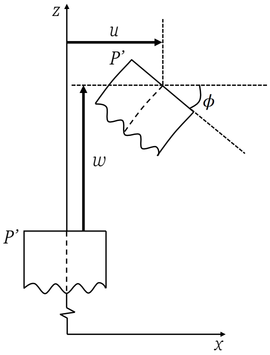
$$\delta\mathcal{U}^s = \iiint_V E\varepsilon\delta\varepsilon dV \quad (6)$$

- From the kinematical hypothesis for the motion of the cross-section (Euler-Bernoulli beam model)

$$w_P = w - x\sin\phi \approx w - xu' \quad (7)$$

$$u_P = u + x(\cos\phi - 1) \approx u \quad (8)$$

$$\phi = \arctan\left(\frac{u'}{1+w'}\right) \approx u' \quad (9)$$



Extracted from Ferreira *et al.* (2022)

Formulation

- We use the quadratic strain as an approximation for the linear strain $\Rightarrow$ Allows including the geometric stiffness in the formulation

$$\varepsilon_P = \frac{1}{2}(u'_P)^2 + w_P + \frac{1}{2}(w'_P)^2 \approx w' + \frac{1}{2}(u')^2 - xu'' \tag{10}$$

- After the integral by parts:

$$\begin{aligned}\delta\mathcal{U}^s = &\left[EA\left(w' + \frac{1}{2}(u')^2\right)\delta w\right]_0^L + \left[EAu'\left(w' + \frac{1}{2}(u')^2\right)\delta u\right]_0^L \\ &+ \left[(EIu'')\,\delta u'\right]_0^L - \left[(EIu'')'\,\delta u\right]_0^L - \int_0^L EA\left(w' + \frac{1}{2}(u')^2\right)\delta w dz \\ &- \int_0^L \left[\left(EAu'\left(w' + \frac{1}{2}(u')^2\right)\right)'\delta u - (EIu'')''\,\delta u\right] dz\end{aligned} \tag{11}$$

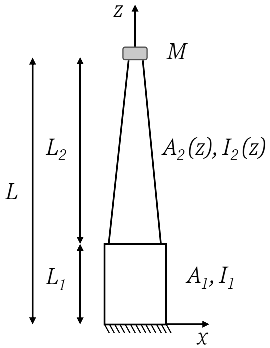
Formulation

- The terms associated with the strain energy due to the SSI represented by a stiffness matrix are

$$\mathcal{U}^K = \frac{1}{2}u^2_{(0)}K_L + u_{(0)}u'_{(0)}K_{LR} + \frac{1}{2}(u'_{(0)})^2K_R \quad (12)$$

$$\delta\mathcal{U}^K = u_{(0)}K_L\delta u_{(0)} + u_{(0)}K_{LR}\delta u'_{(0)} + u'_{(0)}K_{LR}\delta u_0 + u'_{(0)}K_R\delta u'_{(0)} \quad (13)$$

Case I - Fixed base



Extracted from Ferreira *et al.* (2022)

- In this case, the tower is clamped $\Rightarrow$ Null displacements and rotation at $z = 0$ $(u_(0) = 0; u'_(0) = 0)$;
- For simplicity $\hat{\delta}_{(L)} \rightarrow \hat{\delta}(z - L)$
- Subscripts $_{(1)}$ and $_{(2)}$: substructure and tower, respectively.

Case I - Fixed base

- Equation of motion in the axial direction

$$(\rho A + M\hat{\delta}_{(L)})\ddot{w} - \left(EA\left(w' + \frac{1}{2}(u')^2\right)\right)' + (\rho A + M\hat{\delta}_{(L)})g = 0 \tag{14}$$

- Functional form of the transverse equation of motion (valid for any δu):

$$\int_0^L \left[(\rho A + M\hat{\delta}_{(L)})\ddot{u} + (EIu'')'' - \left(EAu'\left(w' + \frac{1}{2}(u')^2\right)\right)'\right]\delta u dz = 0 \tag{15}$$

- Natural boundary conditions:

$$EA_{(L)}\left(w'_{(L)} + \frac{1}{2}(u'_{(L)})^2\right) = 0 \tag{16}$$

$$u''_{(L)} = 0 \tag{17}$$

$$u'''_{(L)} = 0 \tag{18}$$

An approximation

- Due to the difference in the natural frequencies, the axial dynamics usually is disregarded. Hence, we have:

$$-\left(EA\left(w' + \frac{1}{2}(u')^2\right)\right)' + (\rho A + M\hat{\delta}_{(L)})g = 0 \tag{19}$$

- The above equation can be solved after some algebraic work, leading to

$$\left(EA\left(w' + \frac{1}{2}(u')^2\right)\right) = \int_0^z (\rho A + M\hat{\delta}_{(L)})gdz - P_e \tag{20}$$

where $P_e = \int_0^L \rho Agdz + Mg$ is the total weight of the structure.

Equation of motion on the transverse direction

- The inclusion of the axial static response (geometric stiffness) into the transverse equation of motion yields

$$\int_0^L \left((\rho A + M\hat{\delta}_{(L)})\ddot{u} + (EIu'')'' \right) \delta u dz \\ - \int_0^L \left(\left(u'(\rho A + M\hat{\delta}_{(L)})g + u'' \left(\int_0^z (\rho A + M\hat{\delta}_{(L)}) g dz - P_e \right) \right) \right) \delta u dz = 0 \qquad (21)$$

- Solution of the previous equation: separation of variables $u(z,t) = q(t)\psi(x)$ plus Galerkin's method $\delta u = \psi(x)$.

Equation of motion on the transverse direction

- Additional boundary conditions:

$$u_{1(L_1)} = u_{2(L_1)} \tag{22}$$

$$u'_{1(L_1)} = u'_{2(L_1)} \tag{23}$$

$$EI_{1(L_1)}u''_{1(L_1)} = EI_{2(L_1)}u''_{2(L_1)} \tag{24}$$

$$(EI_{1(L_1)}u''_{1(L_1)})' = (EI_{2(L_1)}u''_{2(L_1)})' \tag{25}$$

- Modal shapes for the substructure and the tower:

$$\psi_1(z) = A_1 \sin(\lambda_1 z) + A_2 \cos(\lambda_1 z) + A_3 \sinh(\lambda_1 z) + A_4 \cosh(\lambda_1 z); 0 \leq z \leq L_1 \tag{26}$$

$$\psi_2(z) = A_5 \sin(\lambda_2 z) + A_6 \cos(\lambda_2 z) + A_7 \sinh(\lambda_2 z) + A_8 \cosh(\lambda_2 z); L_1 \leq z \leq L \tag{27}$$

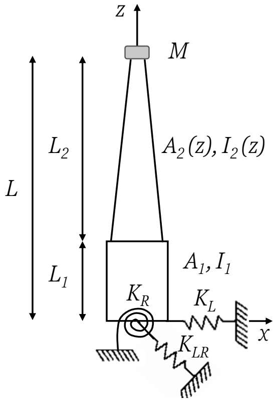
Modal analysis

- Closure $\Rightarrow$ The frequency of the substructure and the tower must be the same, obtained from the dispersion relation from both elements:

$$\lambda_2 = \lambda_1 \left(\frac{EI_1}{m_1} \left(\frac{1}{L - L_1} \int_{L_1}^{L} \frac{m_2}{EI_2} dz \right) \right)^{1/4} \tag{28}$$

- Mode shapes are obtained using the boundary conditions and imposing that the determinant of the coefficient matrix is null (existence of non-trivial solution for the homogeneous problem)

Case II - Elastic base case



- The stiffness matrix at the tip of the substructure appears in the boundary conditions at this point:

$$u'_{(0)}P_e + (EI_{(0)}u''_{(0)})' = -K_L u_{(0)} - K_{LR}u'_{(0)} \quad (29)$$

$$EI_{(0)}u''_{(0)} = K_{LR}u_{(0)} + K_R u'_{(0)} \quad (30)$$

$$u''_{(L)} = 0 \quad (31)$$

$$u'''_{(L)} = 0 \quad (32)$$

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Offshore
Mechanics
Laboratory

Some data from OWTs - Extracted from Ferreira *et al.* (2022)

Input parameter	Blyth	Lely A2	Lely A3	Kentish Flats	Walney 1	Thanet
M [t]	80	32	32	130.8	234.5	130.8
L [m]	71	50	45	76.06	104.6	95.2
L_2 [m]	54.5	37.9	37.9	60.06	67.3	54.1
L_1 [m]	16.5	12.1	7.1	16	37.3	41.1
D_T [m]	2.75	1.9	1.9	2.3	3	2.3
D_B [m]	4.25	3.2	3.2	4.45	5	4.3
e_2 [m]	0.034	0.013	0.013	0.022	0.041	0.036
E_2 [GPa]	210	210	210	210	210	210
D_S [m]	3.5	3.2	3.7	4.3	6	4.7
e_1 [m]	0.050	0.035	0.035	0.045	0.080	0.065
E_1 [GPa]	210	210	210	210	210	210
ρ [kg/m^3]	7860	7860	7860	7860	7860	7860
K_L [GN/m]	42.66	0.52	0.62	0.82	1.53	1.05
K_{LR} [GN]	-45.50	-2.74	-3.57	-5.42	-13.88	-7.89
K_R [GNm/rad]	136.04	23.63	33.59	58.77	205.72	96.84
f_1^{exp} [Hz]	0.488	0.634	0.735	0.339	0.350	0.370

Multimodal expansion

- Multimodal expansion is herein made for the modal analysis

$$
\begin{aligned}
u(z,t) &= q_1(t)\psi_{1,1}(z) + q_2(t)\psi_{2,1}(z) + ... + q_n(t)\psi_{n,1}(z) \;\; for \;\; (0 \leq z \leq L_1) \\
u(z,t) &= q_1(t)\psi_{1,2}(z) + q_2(t)\psi_{2,2}(z) + ... + q_n(t)\psi_{n,2}(z) \;\; for \;\; (L_1 \leq z \leq L)
\end{aligned} \tag{33}
$$

- The modal functions $\psi_{i,j}$ can be independently obtained by applying the approach previously described.

Blyth offshore wind farm. Fixed-base case

N is the number of modes employed in the expansion.

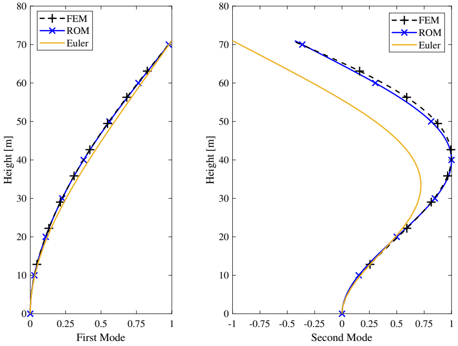
Blyth Offshore Wind Farm											
N	Natural frequency [Hz]						Difference [%]				
	f_1	f_2	f_3	f_4	f_5	f_6	f_1	f_2	f_3	f_4	f_5
1	0.555	-	-	-	-	-	4.72	-	-	-	-
2	0.533	3.827	-	-	-	-	0.57	7.50	-	-	-
3	0.531	3.616	11.285	-	-	-	0.19	1.57	6.96	-	-
4	0.530	3.575	10.782	22.522	-	-	0.00	0.43	2.19	5.55	-
5	0.530	3.566	10.592	21.753	37.907	-	0.00	0.17	0.39	1.94	3.47
6	0.530	3.560	10.551	21.338	36.636	58.120	-	-	-	-	-

Extracted from Ferreira *et al.* (2022)

Natural frequencies ($N = 3$) and the FEM model. Fixed-base case.

Natural frequency	ROM	FEM	Diff. [%]
f_1 [Hz]	0.531	0.525	1.14
f_2 [Hz]	3.616	3.507	3.11

Extracted from Ferreira *et al.* (2022)



Extracted from Ferreira *et al.* (2022)

Blyth offshore wind farm: Elastic-base case

N is the number of modes employed in the expansion.

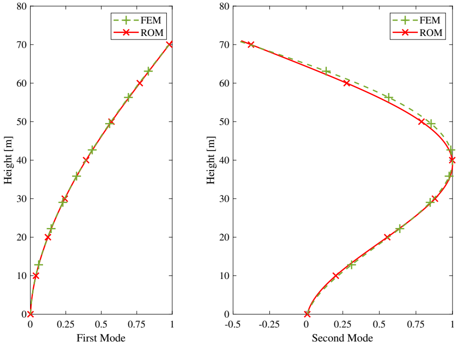
Blyth Offshore Wind Farm											
N	Natural frequency [Hz]						Difference [%]				
	f_1	f_2	f_3	f_4	f_5	f_6	f_1	f_2	f_3	f_4	f_5
1	0.529	-	-	-	-	-	4.34	-	-	-	-
2	0.509	3.629	-	-	-	-	0.39	7.21	-	-	-
3	0.508	3.441	10.631	-	-	-	0.20	1.65	6.54	-	-
4	0.507	3.400	10.206	21.161	-	-	0.00	0.44	2.29	5.34	-
5	0.507	3.391	10.018	20.474	35.841	-	0.00	0.18	0.40	1.92	3.53
6	0.507	3.385	9.978	20.088	34.619	54.906	-	-	-	-	-

Extracted from Ferreira *et al.* (2022)

Natural frequencies ($N = 3$) and the FEM model. Elastic-base case.

Natural frequency	ROM	FEM	Diff. [%]
f_1 [Hz]	0.508	0.502	1.20
f_2 [Hz]	3.441	3.338	3.09

Extracted from Ferreira *et al.* (2022)



Extracted from Ferreira *et al.* (2022)

Natural frequencies: Different OWTs

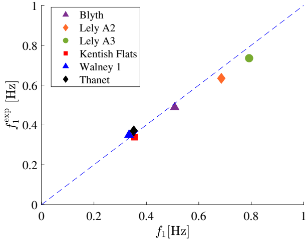
Wind Farm	N	Natural frequency [Hz]	ROM	FEM	Difference [%]
Blyth	3	f_1	0.531	0.525	1.14
		f_2	3.616	3.507	3.11
Lely A2	4	f_1	0.805	0.795	1.26
		f_2	6.337	6.165	2.79
Lely A3	4	f_1	0.890	0.879	1.25
		f_2	7.838	7.569	3.55
Kentish Flats	4	f_1	0.411	0.401	2.49
		f_2	3.389	3.297	2.79
Walney 1	4	f_1	0.389	0.381	2.10
		f_2	2.393	2.352	1.74
Thanet	4	f_1	0.425	0.418	1.67
		f_2	2.238	2.206	1.45

Extracted from Ferreira *et al.* (2022)

Offshore Mechanics Laboratory

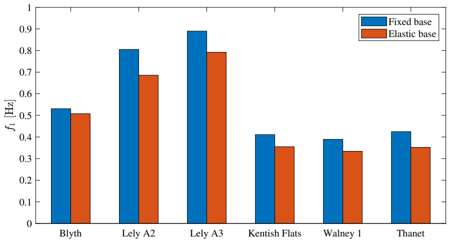
Analytical-experimental correlation (f_1)

Wind Farm	N	ROM [Hz]	Measured [Hz]	Error [%]
Blyth	3	0.508	0.488	4.10
Lely A2	4	0.686	0.634	8.20
Lely A3	4	0.792	0.735	7.76
Kentish Flats	4	0.355	0.339	4.72
Walney 1	4	0.334	0.350	−4.57
Thanet	4	0.352	0.370	−4.86

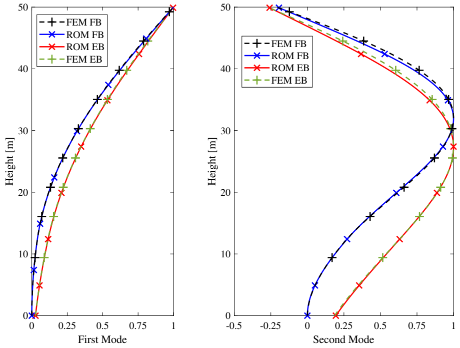


Extracted from Ferreira *et al.* (2022)

Elastic-base *vs.* Fixed-base



Extracted from Ferreira *et al.* (2022)



Extracted from Ferreira *et al.* (2022)

1 Objectives
2 General information
3 Mathematical model
4 Results
5 Final remarks

Offshore Mechanics Laboratory

Final remarks

- The use of FEM is a different approach for assessing the influence of SSI on the natural frequencies ⇒ Higher computational cost;
- Analytical models ⇒ Usually consider an equivalent prismatic tower;
- Analytical models Usually consider 1DoF systems ⇒ Other natural frequencies and modes are not obtained;
- Mode shapes are important from the analysis of strain and stresses;
- The influence of different parameters (for example, those from the foundation) can be easilly assessed by the presented formulation.
- SSI using the foundation model: Decrease the natural frequencies (up to 15% in the investigated towers)

Final remarks

- Model with springs distributed along the monopile ⇒ Also developed by Ynaê in her PhD thesis;
- Model with springs distributed along the monopile ⇒ The application of the Galerkin's method needs attention.
- Model with springs distributed along the monopile ⇒ Paper submitted to Engineering Structures.

Acknowlegments

FAPESP
SÃO PAULO RESEARCH FOUNDATION

CNPq
Conselho Nacional de Desenvolvimento Científico e Tecnológico

CAPES

Office of Naval Research
ONR
Science & Technology

Thank you

gfranzini@usp.br

Offshore Mechanics Laboratory