Analytical models for the mooring stiffness of FOWT's and FOWF's

Workshop on dynamics of wind turbines - Universidade de Brasília

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Objectives

- To present analytical models for evaluating the stiffness associated with the mooring system in floating offshore wind turbines (FOWT's) and floating offshore wind farms (FOWF's);
- To present some results of interest regarding the dynamics of these systems.

PhD Candidate Giovanni Amaral

- Giovanni has worked on the theme since his last semester as a graduate student (2017);
- Strong collaboration with Prof. Celso Pesce;
- 2018-2020: MSc. dissertation entitled "Analytical assessment of the mooring system stiffness" (see this link);
- Results from the MSc. dissertation: Amaral, Pesce & Franzini. Mooring system stiffness: A six-degree-of-freedom closed-form analytical formulation, *Marine Structures*, v. 84, 103189, 2022. The paper can be downloaded from here;
- PhD research: Extension to FOWFs (planar problem) and associated studies.

Offshore Mechanics Laboratory

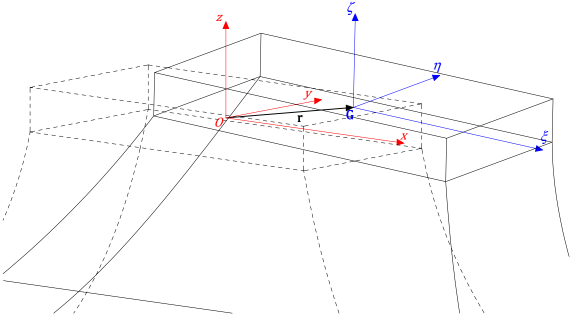
Hypotheses

- The floating unit is a rigid body (In some concepts of FOWTs, this may not hold);
- The forces due to the mooring system come from a potential function $V \Rightarrow$ They only depend on the position and aspects such as friction between line and soil are not considered;
- The forces on the line as function of its geometric configuration must be known *a priori*.

Nomenclature

- $T^{(i)}$: Tension on the i-th mooring line;
- $F_H^{(i)}, F_V^{(i)}$: Vertical and horizontal forces on the i-th mooring line;
- $h_f^{(i)}, v_f^{(i)}$: anchor-fairlead horizontal and vertical distances;
- $\hat{e}_h^{(i)}, \hat{e}_h^{(i)}$: Directional vectors in the vertical and the horizontal directions;
- $\overrightarrow{P}_{E_x}^{(i)}, \overrightarrow{P}_{E_\xi}^{(i)}$: Position of the i-th fairlead with respect to the fixed and the moving frames;
- $\overrightarrow{A}_{E_x}^{(i)}$: Anchor position with respect to the fixed frame.

Problem description



Extracted from Amaral *et al.* (2022) - *Mar. Struc.*

The generalized coordinate vector:

$$\mathbf{q} = \begin{bmatrix} r_x & r_y & r_z & \phi & \theta & \psi \end{bmatrix}^t \qquad (1)$$

where:

$$\mathbf{r} = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}^t \qquad (2)$$

and

$$\theta = \begin{bmatrix} \phi & \theta & \psi \end{bmatrix}^t \qquad (3)$$

Geometric relations

$$\overrightarrow{P}^{(i)}_{E_\xi} = \left(P^{(i)} - G\right) = \begin{bmatrix} p^{(i)}_\xi & p^{(i)}_\eta & p^{(i)}_\zeta \end{bmatrix}^t \quad (4)$$

$$\overrightarrow{P}^{(i)}_{E_x} = \left(P^{(i)} - O\right) = \begin{bmatrix} p^{(i)}_x & p^{(i)}_y & p^{(i)}_z \end{bmatrix}^t = \mathbf{r} + [\mathbb{R}]_{E_x|E_\xi} \overrightarrow{P}^{(i)}_{E_\xi} \quad (5)$$

$$\overrightarrow{A}^{(i)}_{E_x} = \left(A^{(i)} - O\right) = \begin{bmatrix} a^{(i)}_x & a^{(i)}_y & a^{(i)}_z \end{bmatrix}^t \quad (6)$$

$$h^{(i)}_f = \sqrt{\left(a^{(i)}_x - p^{(i)}_x\right)^2 + \left(a^{(i)}_y - p^{(i)}_y\right)^2} \quad (7)$$

$$v^{(i)}_f = p^{(i)}_z - a^{(i)}_z \quad (8)$$

$$\hat{e}^{(i)}_h = \cos\alpha^{(i)}\hat{e}_x + \sin\alpha^{(i)}\hat{e}_y \quad (9)$$

$$\hat{e}^{(i)}_v = -\hat{e}_z \quad (10)$$

where:

$$\cos\alpha^{(i)} = \frac{a^{(i)}_x - p^{(i)}_x}{r^{(i)}} \quad (11)$$

Forces on the mooring lines

- Tension tension on the i-th mooring line:

$$\overrightarrow{T}^{(i)}(h_f^{(i)}, v_f^{(i)}) = F_H^{(i)}(h_f^{(i)}, v_f^{(i)})\hat{e}_h^{(i)} + F_V^{(i)}(h_f^{(i)}, v_f^{(i)})\hat{e}_v^{(i)} \tag{13}$$

where a$F_H^{(i)}$ and $F_V^{(i)}$ are the horizontal and vertical forces components;

- Vector of generalized forces:

$$\mathbf{Q} = \begin{bmatrix} Q_{r_x} & Q_{r_y} & Q_{r_z} & Q_{\phi} & Q_{\theta} & Q_{\psi} \end{bmatrix}^t \tag{14}$$

$$Q_j = \sum_{i=1}^{N} Q_j^{(i)} = \sum_{i=1}^{N} \overrightarrow{T^{(i)}} \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} = \sum_{i=1}^{N} \left(F_H^{(i)} \hat{e}_h^{(i)} + F_V^{(i)} \hat{e}_v^{(i)} \right) \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} \tag{15}$$

Generalized forces

$$Q_{r_x}^{(i)} = F_H^{(i)} \cos\alpha^{(i)} \tag{16}$$

$$Q_{r_y}^{(i)} = F_H^{(i)} \sin\alpha^{(i)} \tag{17}$$

$$Q_{r_z}^{(i)} = -F_V^{(i)} \tag{18}$$

$$Q_{\phi}^{(i)} = F_H^{(i)} \left(\cos\alpha^{(i)} \frac{\partial p_x^{(i)}}{\partial \phi} + \sin\alpha^{(i)} \frac{\partial p_y^{(i)}}{\partial \phi} \right) - F_V^{(i)} \frac{\partial p_z^{(i)}}{\partial \phi} \tag{19}$$

$$Q_{\theta}^{(i)} = F_H^{(i)} \left(\cos\alpha^{(i)} \frac{\partial p_x^{(i)}}{\partial \theta} + \sin\alpha^{(i)} \frac{\partial p_y^{(i)}}{\partial \theta} \right) - F_V^{(i)} \frac{\partial p_z^{(i)}}{\partial \theta} \tag{20}$$

$$Q_{\psi}^{(i)} = F_H^{(i)} \left(\cos\alpha^{(i)} \frac{\partial p_x^{(i)}}{\partial \psi} + \sin\alpha^{(i)} \frac{\partial p_y^{(i)}}{\partial \psi} \right) \tag{21}$$

Using Analytical Mechanics

- The generalized forces: Obtained from the potential function $V = V(\mathbf{q}, \Pi)$, Π are parameters of the mooring system (dimensions, etc):

$$Q_j = -\frac{\partial V}{\partial q_j} \tag{22}$$

- Stiffness matrix $\Rightarrow$ Hessian of the potential energy, evaluated at a position $\mathbf{q}^0$:

$$\mathbb{K}(\mathbf{q}^0) = \left[\frac{\partial^2 V}{\partial q_j \partial q_k}\right]_{\mathbf{q}^0} = -\left[\frac{\partial Q_j}{\partial q_k}\right]_{\mathbf{q}^0} \tag{23}$$

After a lot of Algebraic work

$$K_{jk} = \sum_{i=1}^{N} K_{jk}^{(i)} = -\sum_{i=1}^{N} \left(\frac{\partial F_H^{(i)}}{\partial h_f^{(i)}} \frac{\partial h_f^{(i)}}{\partial q_k} + \frac{\partial F_H^{(i)}}{\partial v_f^{(i)}} \frac{\partial v_f^{(i)}}{\partial q_k} \right) \hat{e}_h^{(i)} \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} +$$
$$- \sum_{i=1}^{N} \left(\frac{\partial F_V^{(i)}}{\partial h_f^{(i)}} \frac{\partial h_f^{(i)}}{\partial q_k} + \frac{\partial F_V^{(i)}}{\partial v_f^{(i)}} \frac{\partial v_f^{(i)}}{\partial q_k} \right) \hat{e}_v^{(i)} \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} +$$
$$- \sum_{i=1}^{N} \left[F_H^{(i)} \frac{\partial}{\partial q_k} \left(\hat{e}_h^{(i)} \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} \right) + F_V^{(i)} \frac{\partial}{\partial q_k} \left(\hat{e}_v^{(i)} \cdot \frac{\partial \overrightarrow{P}^{(i)}}{\partial q_j} \right) \right] \quad (24)$$

with $K_{jk}^{(i)}$ being the stiffness coefficient for the i-th mooring line.

After more algebraic work and considering a perfectly polar symmetry

$$K_{11} = \frac{n}{2}(k_{HH} + \bar{k}_{HH}) \quad (25)$$

$$K_{15} = K_{51} = \frac{n}{2}(k_{VH}l + k_{HH}p_\zeta + \bar{k}_{HH}p_\zeta) \quad (26)$$

$$K_{22} = \frac{n}{2}(k_{HH} + \bar{k}_{HH}) \quad (27)$$

$$K_{24} = K_{42} = -\frac{n}{2}(k_{VH}l + k_{HH}p_\zeta + \bar{k}_{HH}p_\zeta) \quad (28)$$

$$K_{33} = nk_{VV} \quad (29)$$

$$K_{44} = \frac{n}{2}(p_\zeta^2 k_{HH} + p_\zeta^2 \bar{k}_{HH} + 2p_\zeta l k_{HV} + l^2 k_{VV} + lF_H - 2p_\zeta F_V) \quad (30)$$

$$K_{55} = \frac{n}{2}(p_\zeta^2 k_{HH} + p_\zeta^2 \bar{k}_{HH} + 2p_\zeta l k_{HV} + l^2 k_{VV} + lF_H - 2p_\zeta F_V) \quad (31)$$

$$K_{66} = n\bar{k}_{HH}l^2\left(1 - \frac{h}{l}\right) \quad (32)$$

l: radius from the platform vertical central line to the fairleads

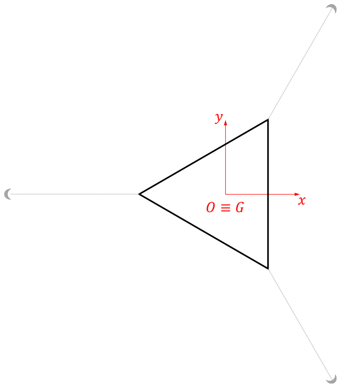
Step-by-step procedure for the mooring system stiffness calculation

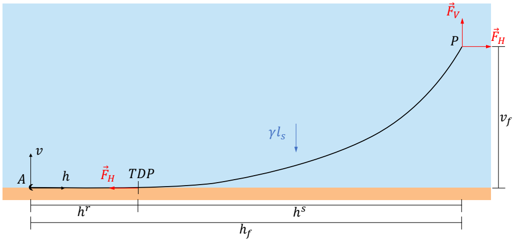
1. Mooring system definition: keypoints, number of lines, line composition
2. Position to be evaluated: $\boldsymbol{r} = [r_x \quad r_y \quad r_z]^t$ and $\boldsymbol{\theta} = [\phi \quad \theta \quad \psi]^t$
3. For the i-th mooring line
   i. Compute $h_f^{(i)}$, $v_f^{(i)}$ and $\alpha^{(i)}$;
   ii. Compute $F_H^{(i)}$, $F_V^{(i)}$, $K_{HH}^{(i)}$, $K_{HV}^{(i)}$, $K_{VH}^{(i)}$ and $K_{VV}^{(i)}$;
   iii. Compute $\mathbb{K}^{(i)}$, such as:

$$\mathbb{K}^{(i)} = \left(\begin{array}{c|c} \mathbb{K}_{TT}^{(i)} & \mathbb{K}_{TR}^{(i)} \\ \hline \mathbb{K}_{RT}^{(i)} & \mathbb{K}_{RR}^{(i)} \end{array}\right)$$

Extracted from Amaral *et al.* (2022) - *Mar. Struc.*

OC4 Mooring System - Extracted from Amaral *et al.* (2022) - *Mar. Struc.*





OC4 Mooring System

Number of mooring lines	3
System type	Spread system
Line profile	One-segment
Line composition	Chain
Water depth	200 m
Fairlead depth	14 m
Radius from center to anchors	834.6 m
Radius from center to fairleads	40.9 m
Unstretched lenght	835.35 m
Mass per unit length	113.35 kg/m
Equivalent diameter	76.6 mm
Axial Stiffness	753.6 MN

Pretensioning (f_{OC4})	1.16E+3 kN
Horizontal force (F_H)	9.63E+2 kN
Vertical force (F_V)	6.49E+2 kN
Horizontal local stiffness (k_{HH})	5.29E+1 kN/m
Vertical local stiffness (k_{VV})	6.84 kN/m
Coupled local stiffness (k_{HV})	1.62E+1 kN/m
Horizontal string stiffness" ($\bar{k}_{HH}$)	1.21 kN/m

Tension functions

- For an extensible "catenary", F_H and F_V are related to h_f and v_f in closed form:

$$h_f = l - \frac{1}{\gamma}F_V + \frac{l}{EA}F_H + \frac{F_H}{\gamma}\ln\left(\frac{F_V + \sqrt{F_H^2 + F_V^2}}{F_H}\right) \quad (33)$$

$$v_f = \frac{1}{\gamma}\sqrt{F_H^2 + F_V^2} + \frac{1}{2}\frac{F_V^2}{EA\gamma} \quad (34)$$

- Newthon-Raphson scheme is adopted for evaluating the stiffness.

Stiffness matrices at neutral position: Analytical and numerical results

$$[\mathbb{K}]_A = \begin{pmatrix} 7.09\text{E+}1 & 0 & 0 & 0 & -1.07\text{E+}2 & 0 \\ 0 & 7.09\text{E+}1 & 0 & 1.07\text{E+}2 & 0 & 0 \\ 0 & 0 & 1.91\text{E+}1 & 0 & 0 & 0 \\ 0 & 1.07\text{E+}2 & 0 & 8.73\text{E+}4 & 0 & 0 \\ -1.07\text{E+}2 & 0 & 0 & 0 & 8.73\text{E+}4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.17\text{E+}5 \end{pmatrix} \quad (35)$$

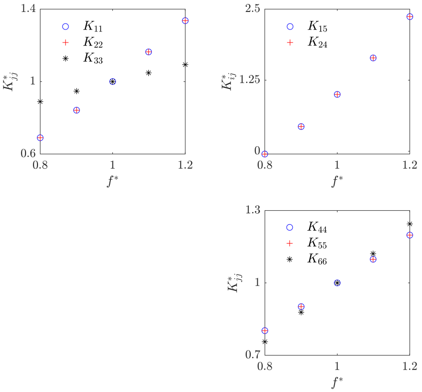
$$[\mathbb{K}]_N = \begin{pmatrix} 7.08\text{E+}1 & 0 & 0 & 0 & -1.08\text{E+}2 & 0 \\ 0 & 7.08\text{E+}1 & 0 & 1.08\text{E+}2 & 0 & 0 \\ 0 & 0 & 1.91\text{E+}1 & 0 & 0 & 0 \\ 0 & 1.07\text{E+}2 & 0 & 8.73\text{E+}4 & 0 & 0 \\ -1.07\text{E+}2 & 0 & 0 & 0 & 8.73\text{E+}4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.17\text{E+}5 \end{pmatrix} \quad (36)$$

Stiffness matrices

- Numerical results: Very small displacements (????) are prescribed and the forces on the floating unit are computed;
- Numerical results: Coefficients of the stiffness matrices are computed by numerical differentiation;
- Numerical results: High computational cost.

- Analytical results: Practically no computational cost;
- Analytical results: Very useull in the early stages of design, in which a large number of conditions must be considered;
- Analytical results: Valid under the modeling hypotheses.

Influence of the pretension



Extracted from Amaral *et al.* (2022) - *Mar. Struc.*

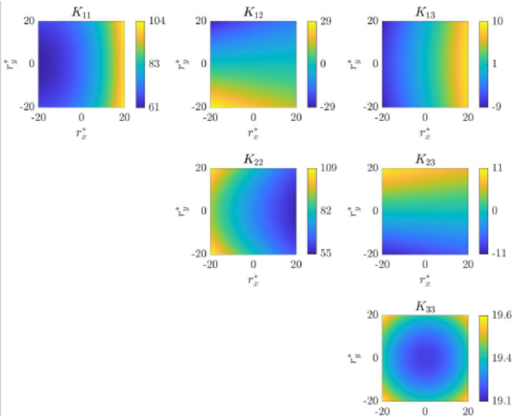
- In the plot at left, both the stiffness and the pretension are normalized with respect to the nominal values;
- These information are relevant in the design stages and can be easily obtained from the analytical formulation.

Non-neutral yaw position

- Consider that a certain load lead to a different static yaw position $\psi = 5^\circ$
- The new stiffness matrix, obtained from the linearization around the static position is

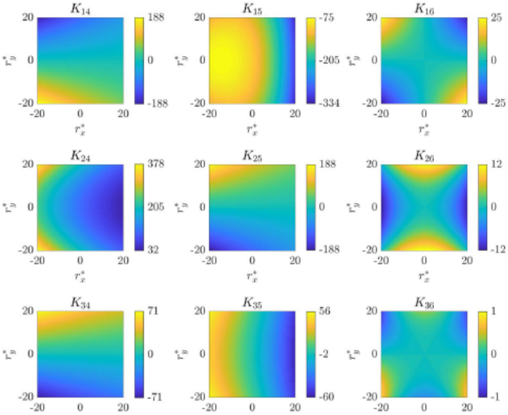
$$[\mathbb{K}]_{\bar{\psi}} = \begin{pmatrix} 7.16\text{E+1} & 0 & 0 & -9.14\text{E+1} & -1.08\text{E+2} & 0 \\ 0 & 7.16\text{E+1} & 0 & 1.08\text{E+2} & -9.14\text{E+4} & 0 \\ 0 & 0 & 1.92\text{E+1} & 0 & 0 & 1.63\text{E+2} \\ -9.14\text{E+1} & 1.08\text{E+2} & 0 & 8.77\text{E+4} & -5.14\text{E+3} & 0 \\ -1.08\text{E+2} & -9.14\text{E+4} & 0 & -5.14\text{E+3} & 8.77\text{E+4} & 0 \\ 0 & 0 & 1.63\text{E+2} & 0 & 0 & 1.19\text{E+5} \end{pmatrix} \quad (37)$$

Stiffness coefficients at different positions - Amaral *et al.* (2022) - *Mar. Struc.*



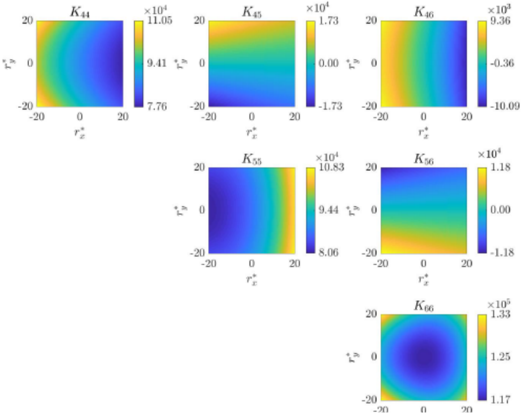
- Coefficients $K_{11}, K_{12}, K_{13}, K_{22}, K_{23}, K_{33}$ as functions of r_x^* and r_y^*;
- $r_{x,y}^*$: Surge/Sway displacements, normalized with respect to the fairlead radius;
- Unities: kN, kNm and rad.

Stiffness coefficients at different positions - Amaral *et al.* (2022) - *Mar. Struc.*



- Coefficients $K_{14}, K_{15}, K_{16}, K_{24}, K_{25}, K_{26}$ as functions of r_x^* and r_y^*;
- $r_{x,y}^*$: Surge/Sway displacements, normalized with respect to the fairlead radius;
- Unities: kN, kNm and rad.

Stiffness coefficients at different positions - Amaral *et al.* (2022) - *Mar. Struc.*



- Coefficients $K_{44}, K_{45}, K_{46}, K_{55}, K_{56}, K_{66}$ as functions of r_x^* and r_y^*;
- $r_{x,y}^*$: Surge/Sway displacements, normalized with respect to the fairlead radius;
- Unities: kN, kNm and rad.

Free vibrations - Undamped natural periods and mode shapes

- In the formulation herein, the mooring system accounts only for the stiffness matrix;
- Damping effects $\Rightarrow$ can be included using Rayleigh's model;
- Mass matrix: The added inertia matrix must be added to the "physical" mass matrix;
- Added inertia matrix $\Rightarrow$ depend on the wave period. For the motions on the horizontal plane (more affected by the mooring system), the results are obtained considering $T \rightarrow \infty$.

Mass (m)	1.3473E+7 kg
Platform yaw inertia about CM ($I_{\psi\psi}$)	1.226E+10 kgm^2
Surge-Surge added mass ($M_{a\xi\xi}$)	6.49E+6 kg
Sway-Sway added mass ($M_{a\eta\eta}$)	6.49E+6 kg
Yaw-Yaw added mass ($M_{a\psi\psi}$)	4.87E+9 kgm^2

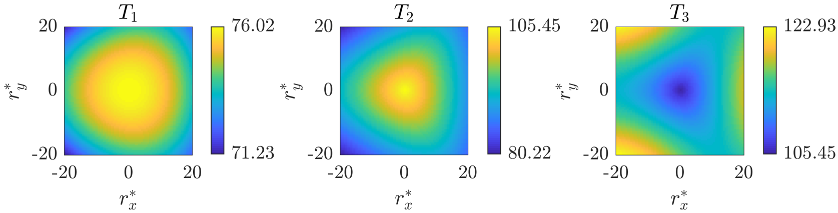
Free vibrations - First three natural periods [s]

	T_1	T_2	T_3
Analytical formulation	76.02	105.48	105.48
Numerical results [Robertson *et al*. (2014)]	76.03	105.53	105.53

- The above results are obtained at the neutral condition;
- The natural periods T_4, T_5, T_6 are those from the motions on the vertical plane;
- First-order loads: In the wave periods ($3 \leq T \leq 30$ s);
- Second-order loads: Observed only in irregular seas;
- Second-order loads: Depend on hydrodynamic coefficients and may excite the periods on the horizontal plane (slow-drift).

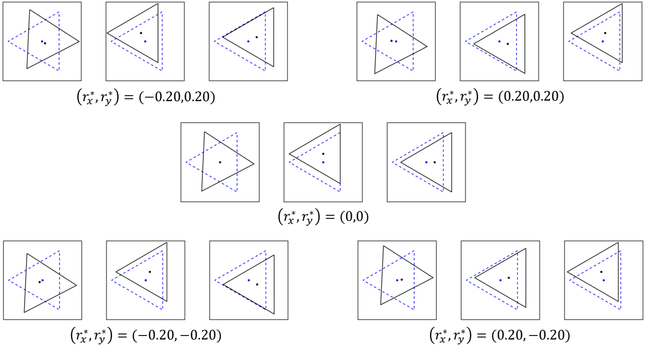
Free vibrations - First three natural periods [s]

- The periods can be obtained as functions of the offset caused, for example, by an enviromental load.



Extracted from Amaral *et al*. (2022) - *Mar. Struc.*

Free vibrations - Modes on the horizontal plane



Extracted from Amaral *et al.* (2022) - *Mar. Struc.*

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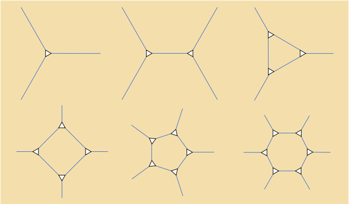
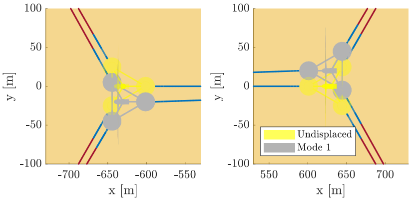
Contextualization

- When the water depth increases, the cost of the mooring system increases ⇒ Shared mooring systems may be interesting for reducing costs;
- Results obtained by Giovanni as part of his PhD thesis ⇒ Extension of the formulation previously presented. Only the planar problem is herein considered;
- The same hypotheses previously adopted are considered;
- Hereafter, all the results are extracted from Giovanni's qualifying written report - Amaral, G.A. *"Analytical tools for design and analysis of shared mooring systems for floating wind farms"*, Escola Politécnica. The exam will take place by the end of the first semester of 2023.

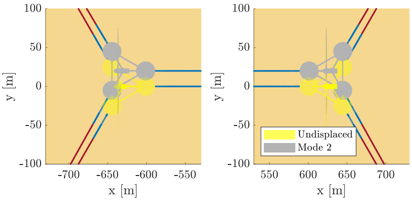


Extracted from Amaral (2023).

Modal analysis at the trivial condition (unloaded system)

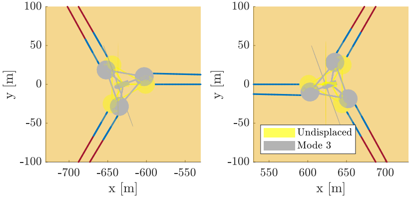


$T_1 = 67.97$ s.

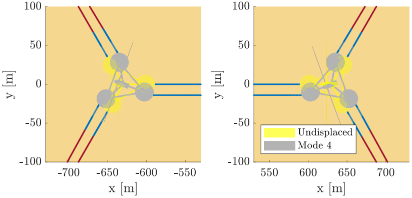


$T_2 = 68.25$ s.

Modal analysis at the trivial condition (unloaded system)

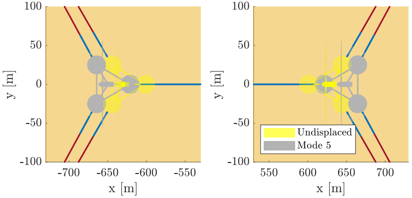


$$T_3 = 81.90 \text{ s.}$$

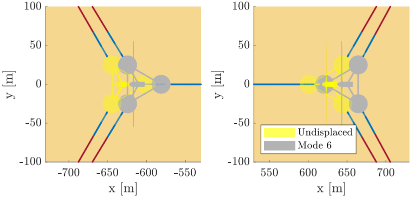


$$T_4 = 82.81 \text{ s.}$$

Modal analysis at the trivial condition (unloaded system)

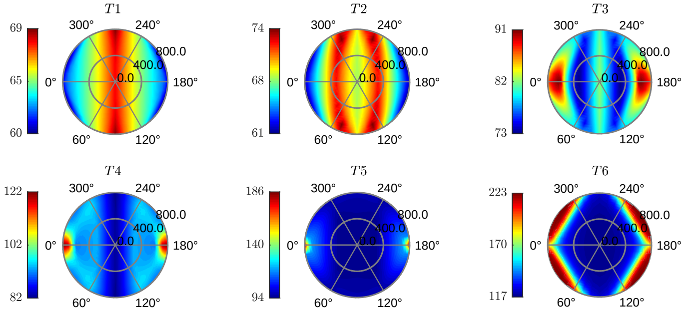


$T_5 = 94.22$ s.



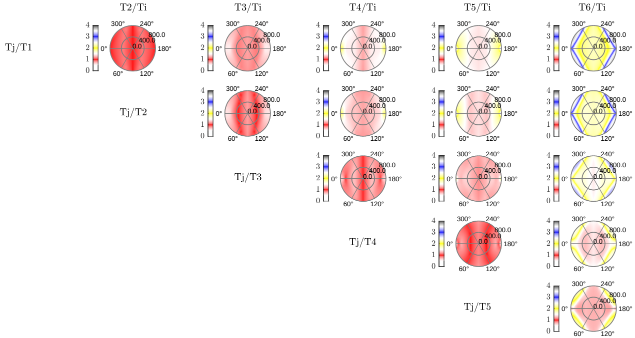
$T_6 = 116.75$ s.

Natural periods as functions of the average force and incidence



Allows a "big picture" of scenarios in a single plot.

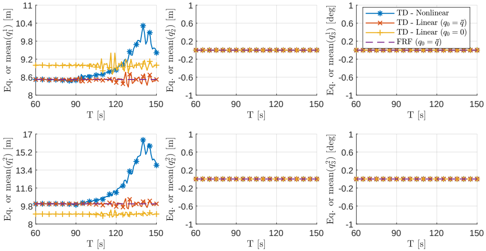
Possibility of internal resonances



- Internal resonances of type T_m/T_n being a rational number can be investigated $\Rightarrow$ Consequences are under study!!!!!

$\bar{F} = 520$ kN, $\tilde{F} = 100$ kN, and $\alpha = 0$ deg

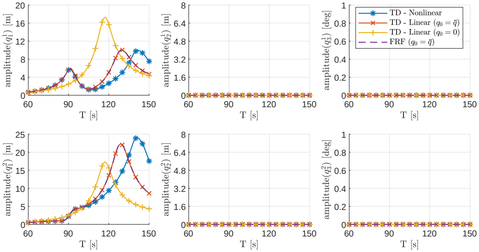
Forces in the x direction are applied to both platforms: $F(t) = \bar{F} + \tilde{F}\sin(2\pi/Tt)$



- Nonlinear time-domain method leads to equilibrium position larger than the linear results $\Rightarrow$ Under investigation, but this point as been obtained by using the MMTS to the Helmholtz-Duffing 1DoF oscillator;

$\bar{F} = 520$ kN, $\tilde{F} = 100$ kN, and $\alpha = 0$ deg

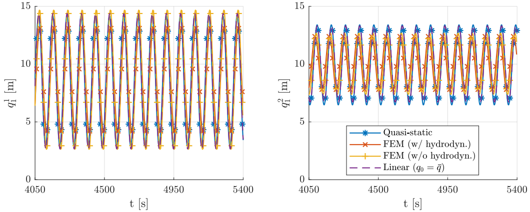
Forces in the x direction are applied to both platforms: : $F(t) = \bar{F} + \tilde{F}\sin(2\pi/Tt)$



- Time-domain method using the stiffness matrix at the trivial condition $\Rightarrow$ Important errors;
- Time-domain method using the "true" stiffness matrix $\Rightarrow$ In agreement with the analytical solution from FRFs;
- Time-domain method using the full nonlinear stiffness forces $\Rightarrow$ Good agreement with the linear results for low-period forcing. Differences observed for $T > 100$ s.

$\bar{F} = 520$ kN, $\tilde{F} = 100$ kN, $T = 92$ s and $\alpha = 0$ deg

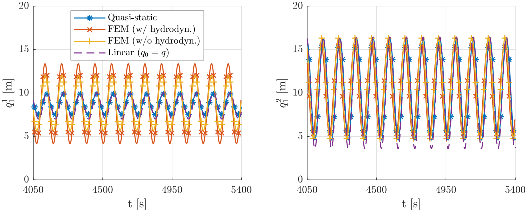
Forces in the x direction are applied to both platform: $F(t) = \bar{F} + \tilde{F}\sin(2\pi/Tt)$



- In this case, the different methodologies do not reveal important differences;

$\bar{F} = 520$ kN, $\tilde{F} = 100$ kN, $T = 108$ s and $\alpha = 0$ deg

Forces in the x direction are applied to both platform: $F(t) = \bar{F} + \tilde{F}\sin(2\pi/Tt)$



- Important differences appear;
- The nonlinear (quadratic) hydrodynamic damping the appears in the mooring linear is not incorporated into the quasi-static nonlinear mooring model;

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Final remarks

- Analytical models for the mooring system stiffness were/are beign developed;
- Analytical models: Fully compared with FEM models for the 1 platform case. For FOWFs, the first results indicate very good agreement in terms of natural periods;
- Analytical models: A number of scenarios can be analysed in a fraction of seconds. The same analyses can take hours in FEM codes;
- Analytical models: Very useful in the design and analyses of FOWT/FOWFs ⇒ Implemented in a Matlab® code for these purposes (SuSSA - SubSea Systems Analysis);
- Environmental forcing models: We are trying to improve them for enhancing SuSSA

Final remarks

- Analytical models: Allowed for optimization of the mooring configuration ⇒ Giovanni did this for his qualifying exam;
- Most of the works on the literature focus on higher-order hierarchical models for the mooring line;
- Usually, the stiffness matrix and natural periods are calculated only on the trivial position ⇒ They can significantly vary with the mean forces (and the corresponding offsets);
- Usually, the design of mooring systems consider static limits ⇒ The methodology under development can assess dynamic responses with low computational cost;
- Higher-order hierarchical models: Important for accounting cases not covered by the hypotheses of the analytical models ⇒ Run selected scenarios for more detailed analysis.

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Thank you

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