



SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS

São Paulo, July 29th - August 9th, 2019

# A REDUCED ORDER MODEL IN OCEAN ENGINEERING DYNAMICS

Module 28

**[Professor Celso P. Pesce](#)**

[ceppesce@usp.br](mailto:ceppesce@usp.br)

University of São Paulo

*with collaboration of*

Dr. Guilherme R. Franzini, Associate Prof.

Dr. Renato M.M. Orsino, Assistant Prof.



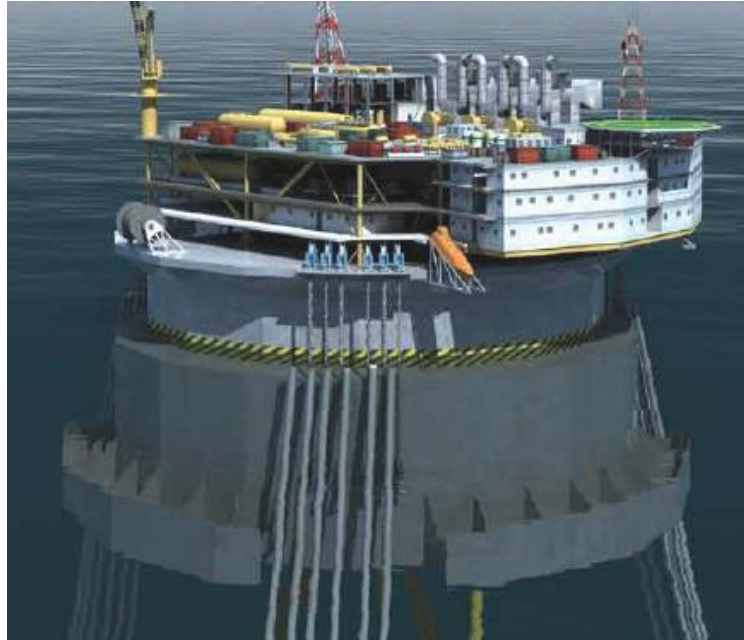
ESCOLA POLITÉCNICA DA  
UNIVERSIDADE DE SÃO PAULO

**FAPESP**

# SUMMARY

1. Analytical Model for Mooring forces on the Horizontal Plane
2. Current Induced Motions of a moored mono-column platform

# MONOCOLUMN PLATFORMS

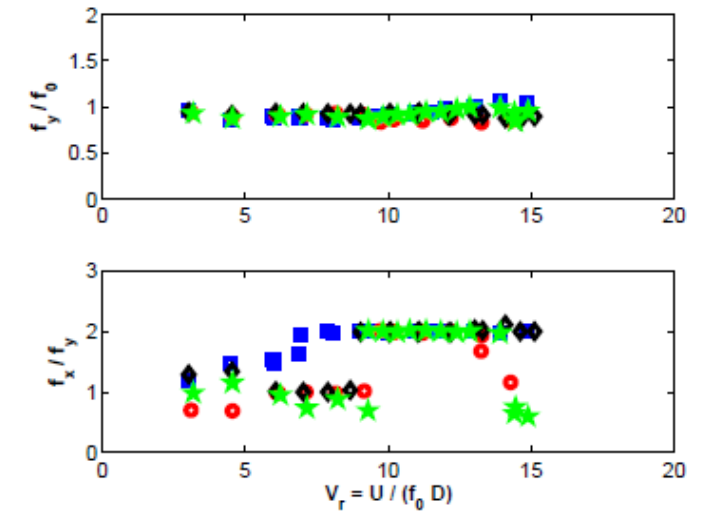
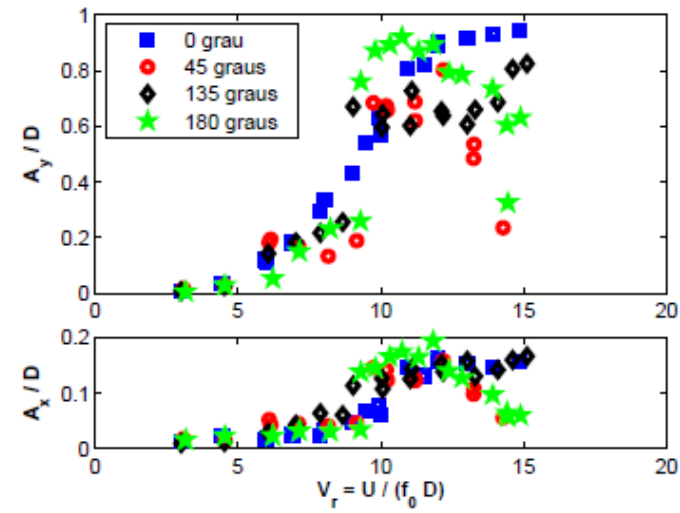
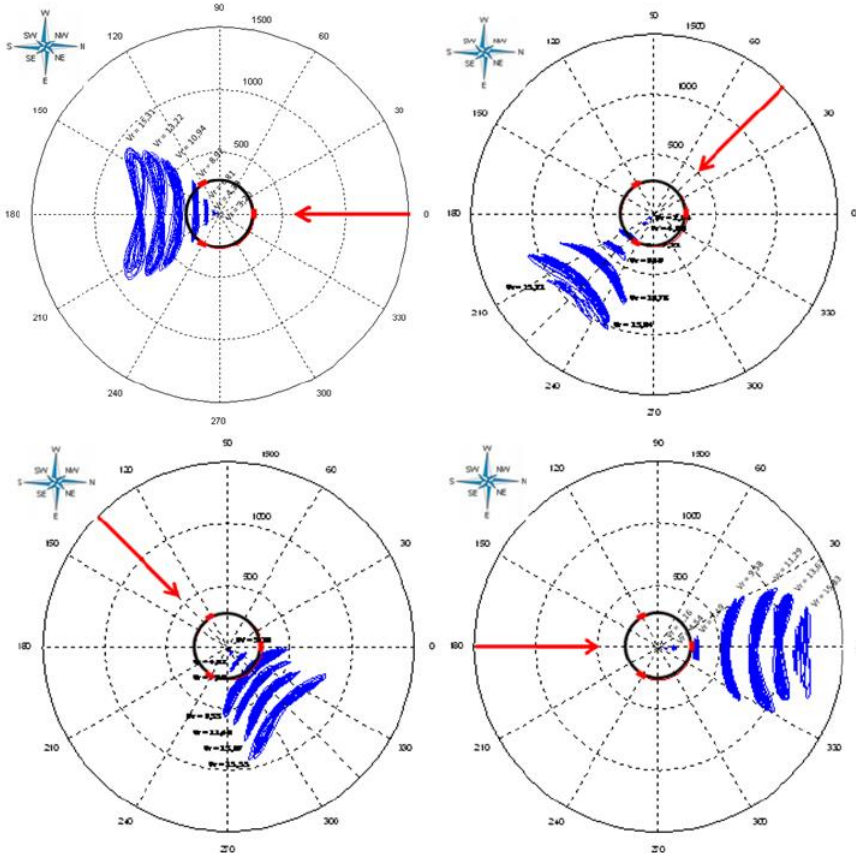


MonoBR-GoM  
with a moon-pool  
(Nishimoto et al, 2010)



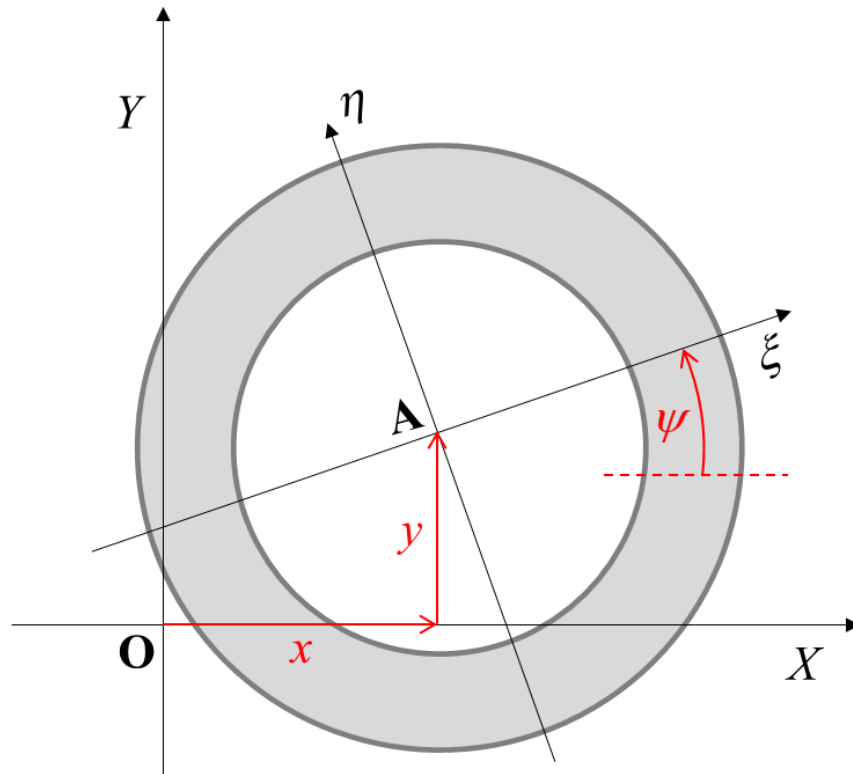
Sevan Marine Co.  
Up: FPSO Sevan Hummingbird.  
Bottom: FPSO Sevan Piranema  
(Gonçalves et al, 2010b)

# MONOBR – TRAJECTORIES AND DYNAMIC RESPONSE



MonoBR, Experimental data  
 Gonçalves, R.T., PhD Thesis, 2013;  
 Gonçalves et al 2010, J Offshore Mech and Arctic Engineering

# REDUCED ORDER MODEL ON THE HORIZONTAL PLANE



$$A \equiv G$$

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial T}{\partial \mathbf{q}} = \mathbf{Q}^m + \mathbf{Q}^v \quad \text{Usual Lagrange Equations}$$

$$\mathbf{q} = [x \quad y \quad \psi]^T \quad \text{Generalized coordinates}$$

$$\mathbf{Q}^m = [Q_j^m] \quad \text{Mooring system generalized forces}$$

$$\mathbf{Q}^v = [Q_j^v]; \quad \text{Generalized viscous hydrodynamic forces due to the incident current}$$

$$j = 1, 2, 3$$



# EQUATIONS OF MOTION

$$T = \frac{1}{2} \dot{\mathbf{q}}^T \mathbf{M} \dot{\mathbf{q}}$$

In the general case, the mass matrix is a full matrix, such that the generalized coordinates couple inertially. In this case, however,

$$\mathbf{M} = \mathbf{M}_p + \mathbf{M}_a = \begin{bmatrix} M_p & 0 & 0 \\ 0 & M_p & 0 \\ 0 & 0 & I_p \end{bmatrix} + \begin{bmatrix} M_a & 0 & 0 \\ 0 & M_a & 0 \\ 0 & 0 & I_a \end{bmatrix}$$

Assuming the body axisymmetric with respect to a vertical axis,  $Gz$ , so that  $\mathbf{M}$  is invariant w.r.t. the rotation  $\psi$

$$M_a = C_a \left( \rho \frac{\pi D^2}{4} H \right) + M_{mp}; \quad I_a = 0$$

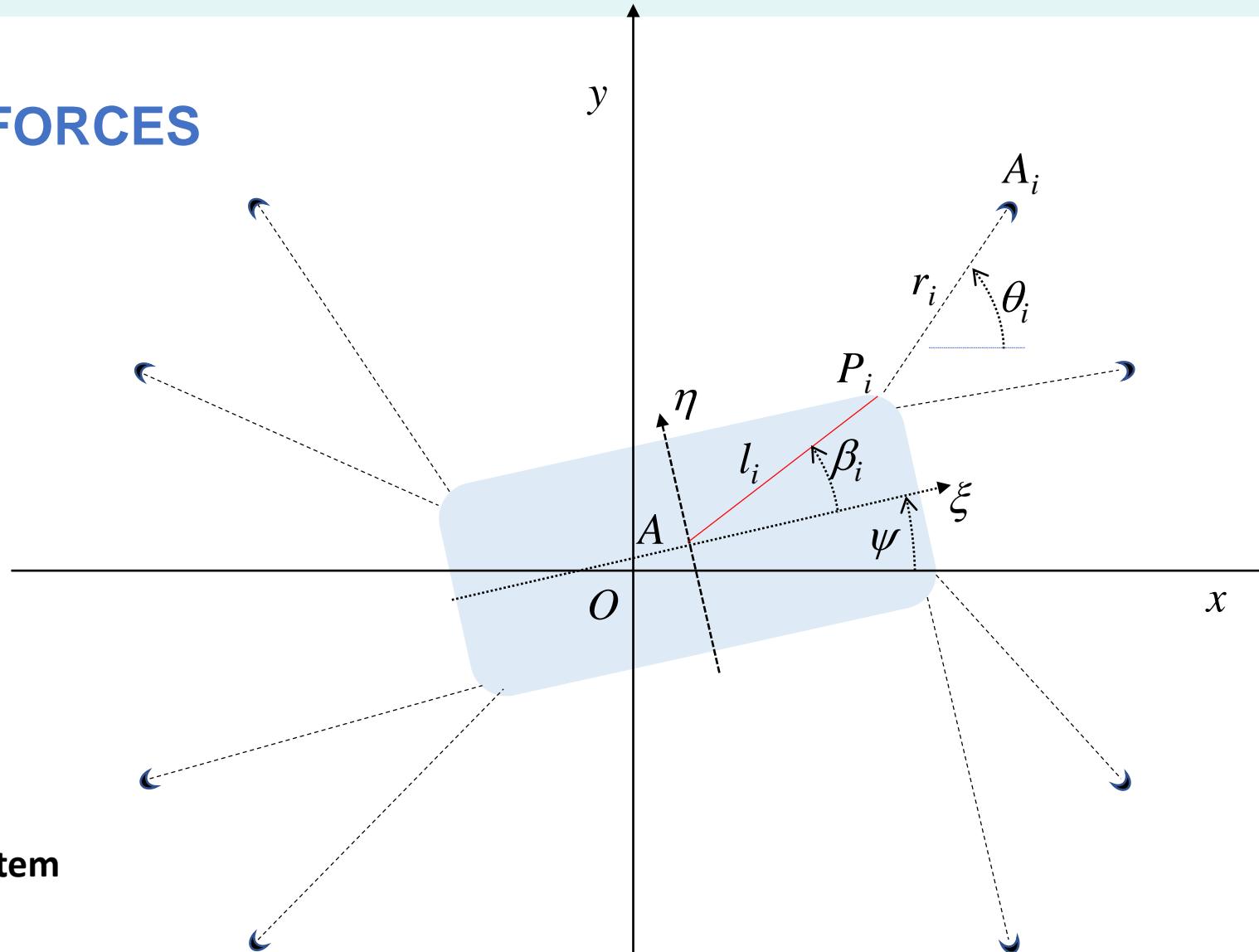
Added mass coefficients at very low frequencies

$M_{mp}$  Mass of water inside the moon-pool is **considered constant**

Neglecting second-order inertial terms

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\mathbf{q}}} \right) - \frac{\partial T}{\partial \mathbf{q}} = \mathbf{Q}^m + \mathbf{Q}^v \quad \longrightarrow \quad \boxed{\mathbf{M} \ddot{\mathbf{q}} = \mathbf{Q}^m + \mathbf{Q}^v}$$

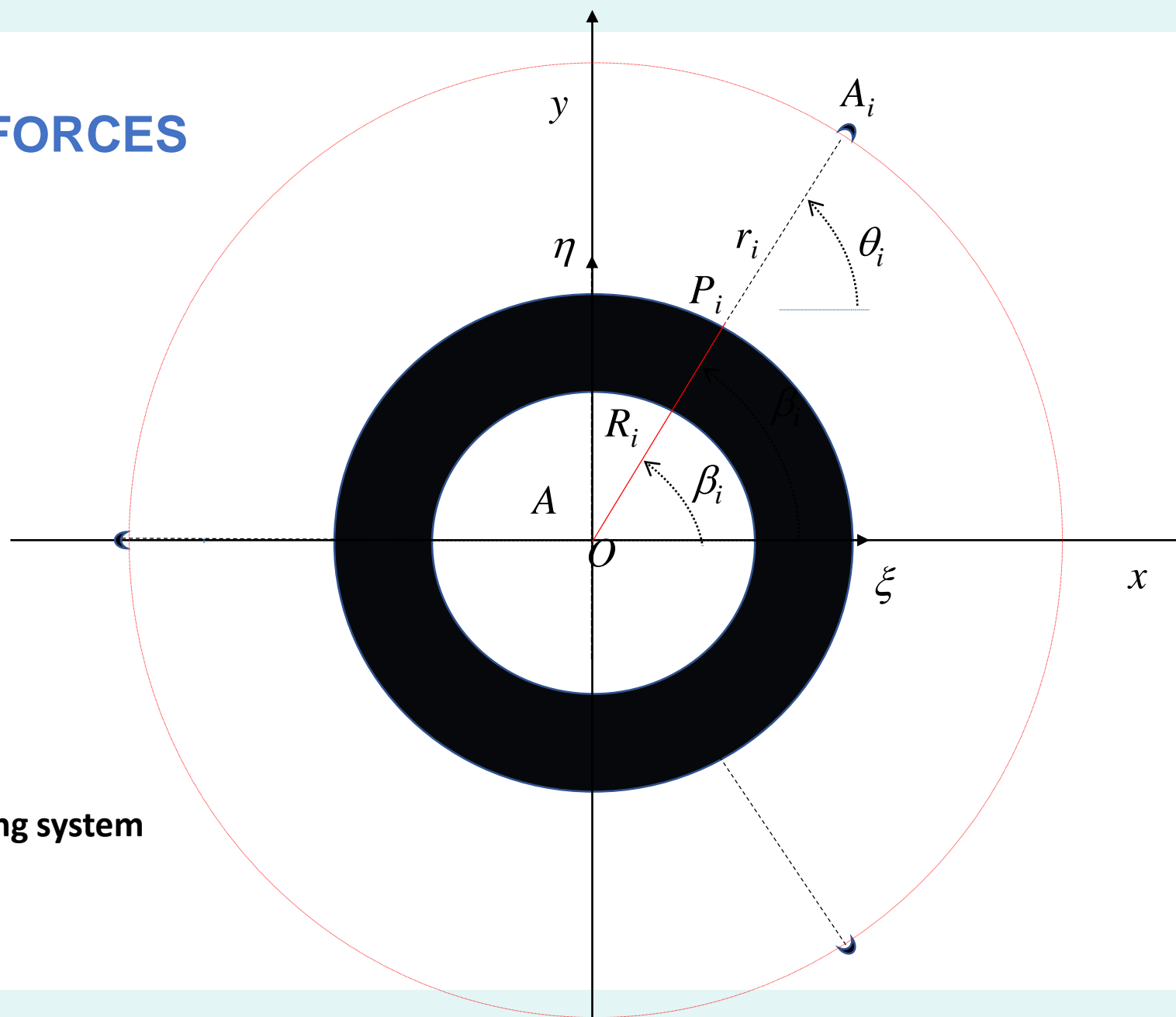
# MOORING FORCES



Generic mooring system

# MOORING FORCES

Horizontal plane  
projection

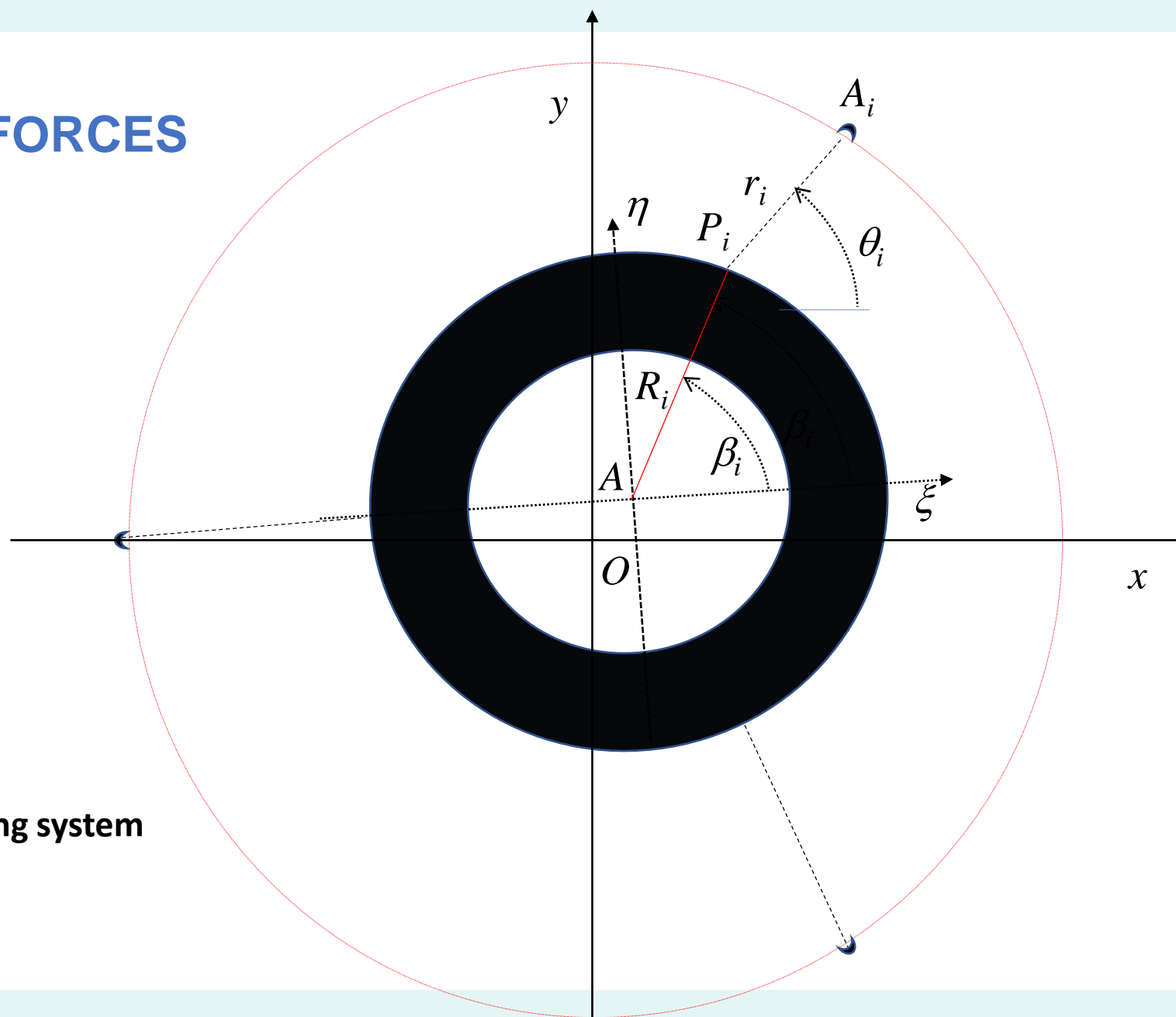


Monocolumn mooring system



# MOORING FORCES

Horizontal plane  
projection



Monocolumn mooring system



# GENERALIZED NONLINEAR MOORING FORCES ON THE HORIZONTAL PLANE

$$Q_j^m = \sum_{i=1}^N \mathbf{F}_i^T \frac{\partial \mathbf{P}_i}{\partial q_j} = \sum_{i=1}^N f_i(r_i) \mathbf{e}_i^T \frac{\partial \mathbf{P}_i}{\partial q_j}; \quad j = 1, 2, 3; \quad i = 1, \dots, N$$

$$\mathbf{F}_i = \mathbf{F}_i(r_i) = f_i(r_i) \mathbf{e}_i; \quad i = 1, \dots, N$$

Horizontal mooring line force function

$$\mathbf{e}_i = \frac{(\mathbf{A}_i - \mathbf{P}_i)}{|\mathbf{A}_i - \mathbf{P}_i|} = \frac{(\mathbf{A}_i - \mathbf{P}_i)}{r_i} = [\cos \theta_i \quad \sin \theta_i]^T; \quad i = 1, \dots, N$$

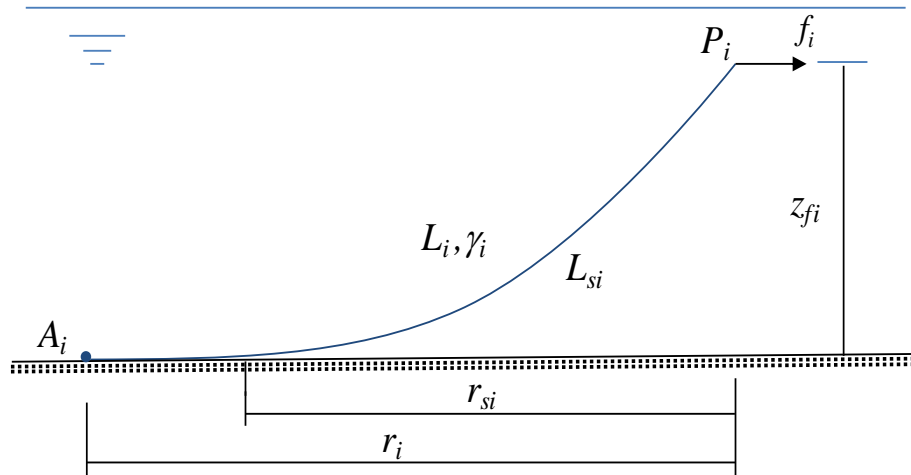
Mooring line unit director vector

$$\mathbf{P}_i = [x + R_i \cos(\psi + \beta_i) \quad y + R_i \sin(\psi + \beta_i)]^T$$

Fairlead position, for each mooring line

# CATENARY MOORING LINES ON A FRICTIONLESS SEABED

$$\frac{r_i(f_i)}{L_i} = 1 - \frac{f_i}{\gamma_i L_i} \left[ \left( \frac{1 + 2 f_i / \gamma_i z_{fi}}{(f_i / \gamma_i z_{fi})^2} \right)^{\frac{1}{2}} - \ln \left( 1 + \frac{\gamma_i z_{fi}}{f_i} + \left( \frac{1 + 2 f_i / \gamma_i z_{fi}}{(f_i / \gamma_i z_{fi})^2} \right)^{\frac{1}{2}} \right) \right]$$



- $\gamma_i$  Mooring line linear immersed weight
- $L_i$  Mooring line total length
- $z_{fi}$  Distance from fairlead to sea bottom

# GENERALIZED LINEAR MOORING FORCES: LOCAL STIFFNESS MATRIX



SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS

$$\mathbf{Q}^m = \mathbf{Q}^m(\mathbf{q}; \Pi) \quad \text{Generalized mooring forces}$$

$$\mathbf{q} = [x \quad y \quad \psi]^T \quad \text{Generalized coordinates}$$

$$\Pi = \{(\mathbf{A}_i \quad R_i \quad \beta_i); i = 1, \dots, N\} \quad \text{Geometric parameters}$$

## LOCAL STIFFNESS MATRIX

$$\mathbf{K}(\mathbf{q}; \Pi, \bar{\mathbf{q}}) = - \left[ \frac{\partial Q_j}{\partial q_k} \right]_{\mathbf{q}=\bar{\mathbf{q}}}$$



$$\mathbf{K}(\mathbf{q}; \Pi, \bar{\mathbf{q}}) = \begin{bmatrix} k_{xx} & k_{xy} & k_{x\psi} \\ k_{xy} & k_{yy} & k_{y\psi} \\ k_{x\psi} & k_{y\psi} & k_{\psi\psi} \end{bmatrix}_{(\bar{x}, \bar{y}, \bar{\psi})}$$

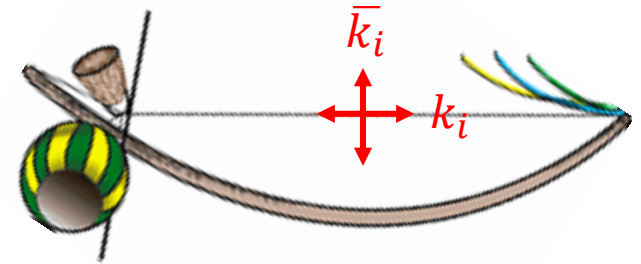


# GENERALIZED LINEAR MOORING FORCES: LOCAL STIFFNESS MATRIX



SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS

$$\begin{aligned}
 K_{jk} &= -\frac{\partial}{\partial q_k} \sum_{i=1}^N f_i(r_i) \vec{e}_i \cdot \frac{\partial P_i}{\partial q_j} = \\
 &= -\sum_{i=1}^N \frac{\partial f_i}{\partial q_k} \vec{e}_i \cdot \frac{\partial P_i}{\partial q_j} - \sum_{i=1}^N f_i \frac{\partial}{\partial q_k} \left( \vec{e}_i \cdot \frac{\partial P_i}{\partial q_j} \right) = \\
 &= -\sum_{i=1}^N f_i' \frac{\partial r_i}{\partial q_k} \vec{e}_i \cdot \frac{\partial P_i}{\partial q_j} - \sum_{i=1}^N f_i \left( \frac{\partial \vec{e}_i}{\partial q_k} \cdot \frac{\partial P_i}{\partial q_j} + \vec{e}_i \cdot \frac{\partial^2 P_i}{\partial q_k \partial q_j} \right)
 \end{aligned}$$



Mooring line  
tangent stiffness

Mooring line  
string stiffness

$$f_i' = df_i/dr_i \equiv k_i(r_i)$$

$$f_i(r_i)/r_i \equiv \bar{k}_i(r_i)$$



# GENERALIZED LINEAR MOORING FORCES: LOCAL STIFFNESS MATRIX

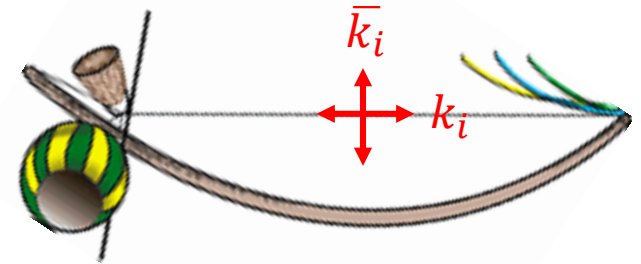


SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS

$$k_{xx} = \sum_{i=1}^N \left( k_i \cos^2(\theta_i) + \bar{k}_i \sin^2(\theta_i) \right)$$

$$k_{yy} = \sum_{i=1}^N \left( k_i \sin^2(\theta_i) + \bar{k}_i \cos^2(\theta_i) \right)$$

$$k_{\psi\psi} = \sum_{i=1}^N \left( k_i R_i^2 \sin^2(\psi + \beta_i - \theta_i) \right) + \sum_{i=1}^N \bar{k}_i R_i^2 \left( \cos^2(\psi + \beta_i - \theta_i) + \frac{r_i}{R_i} \cos(\psi + \beta_i - \theta_i) \right)$$

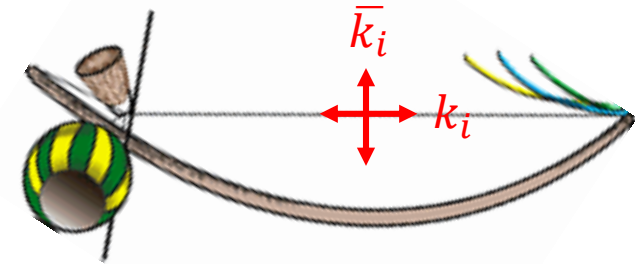


# GENERALIZED LINEAR MOORING FORCES: LOCAL STIFFNESS MATRIX



SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS

$$k_{xy} = k_{yx} = \sum_{i=1}^N (k_i - \bar{k}_i) \sin(\theta_i) \cos(\theta_i)$$

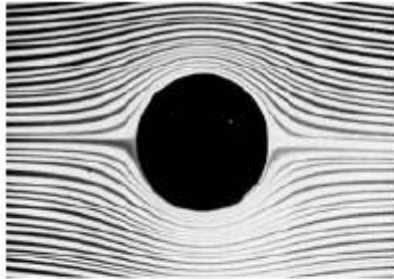


$$k_{x\psi} = k_{\psi x} = - \sum_{i=1}^N (k_i R_i \cos(\theta_i) \sin(\psi + \beta_i - \theta_i)) - \sum_{i=1}^N (\bar{k}_i R_i \sin(\theta_i) \cos(\psi$$

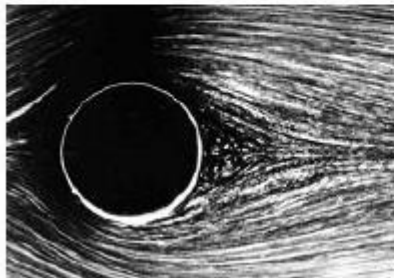
$$k_{y\psi} = k_{\psi y} = - \sum_{i=1}^N (k_i R_i \sin(\theta_i) \sin(\psi + \beta_i - \theta_i)) + \sum_{i=1}^N (\bar{k}_i R_i \cos(\theta_i) \cos(\psi + \beta_i - \theta_i))$$



# BASICS ON CLASSIC VIV



Sem separação

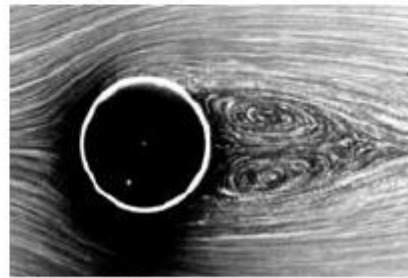


Re = 9,6



Van Dyke, 1982

Re = 13



Re = 26

Meneghini et al, 2010

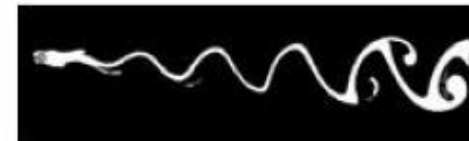


Batchelor, 1967

Re = 32



Re = 55



Re = 65



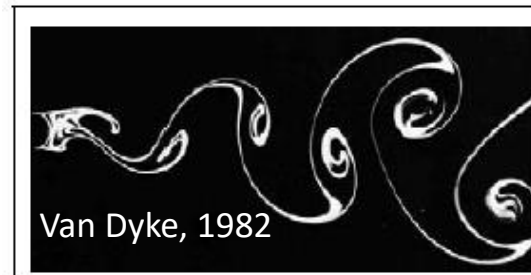
Re = 73



Re = 102

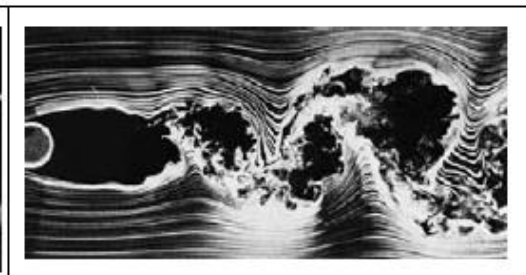


Re = 161



Van Dyke, 1982

Re=140



Re=10<sup>4</sup>



# CLASSICAL VIV

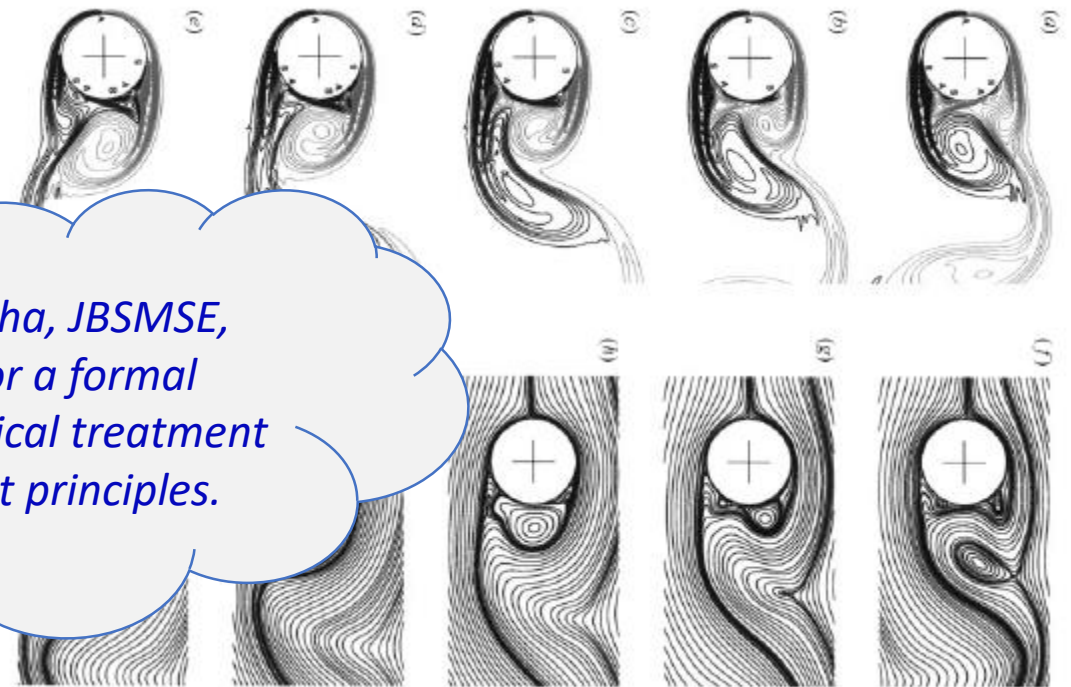
## Vortex shedding:

- ✓ Flow pasting bluff-bodies
- ✓ Comes from the inherent instability and interaction between shear layers
- ✓ Is a self-regulated and stable phenomenon: **Hopf bifurcation** with onset at  $Re \sim 50$
- ✓ Important dimensionless parameters:
  - $Re = UD/\nu$  Reynolds number
  - $St = f_s D/U$  Strouhal number
- ✓ Reynolds number: ratio between inertial and viscous forces
- ✓ Strouhal number: depends on the body shape and regulates the shedding frequency

*Vortex-wake dynamics could be represented through van der Pol or Rayleigh oscillators*

*See Aranha, JBSMSE, 2004 for a formal mathematical treatment from first principles.*

Vorticity contours



Generation and vortex shedding at  $Re=500$ . Half-cycle.  
Blackburn & Henderson, 1999

# CLASSICAL VIV

## Vortex Induced Vibrations:

- ✓ Nonlinear fluid-structure interaction resonant phenomenon
- ✓ Self-regulated
- ✓ Important dimensionless parameters:

$U^* = V_r = U / f_n D$	Reduced velocity
$f_s^* = f_s / f_n = StU^*$	Shedding frequency
$f^* = f / f_n$	Response frequency
$m^* = m / m_d$	Mass ratio (specific density)
$C_a = m_a / m_d$	Added mass coefficient
$\zeta = c / 2m\omega_n = c / 4\pi m f_n$	Structural damping

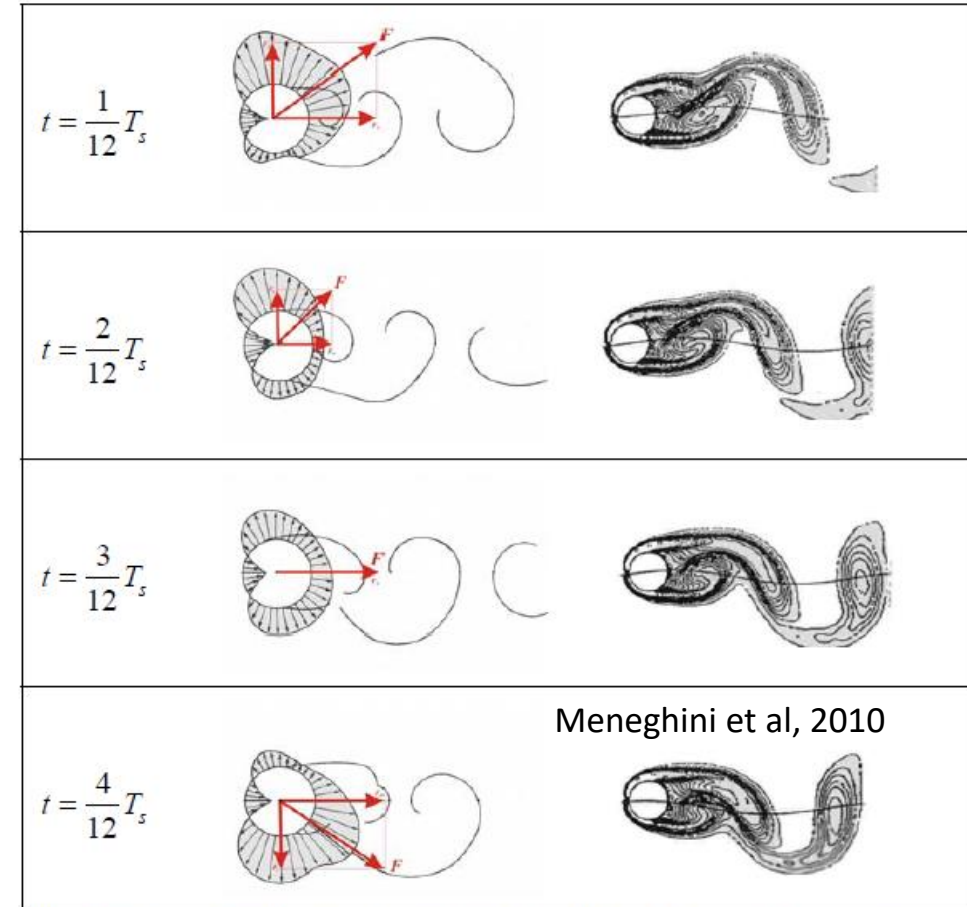
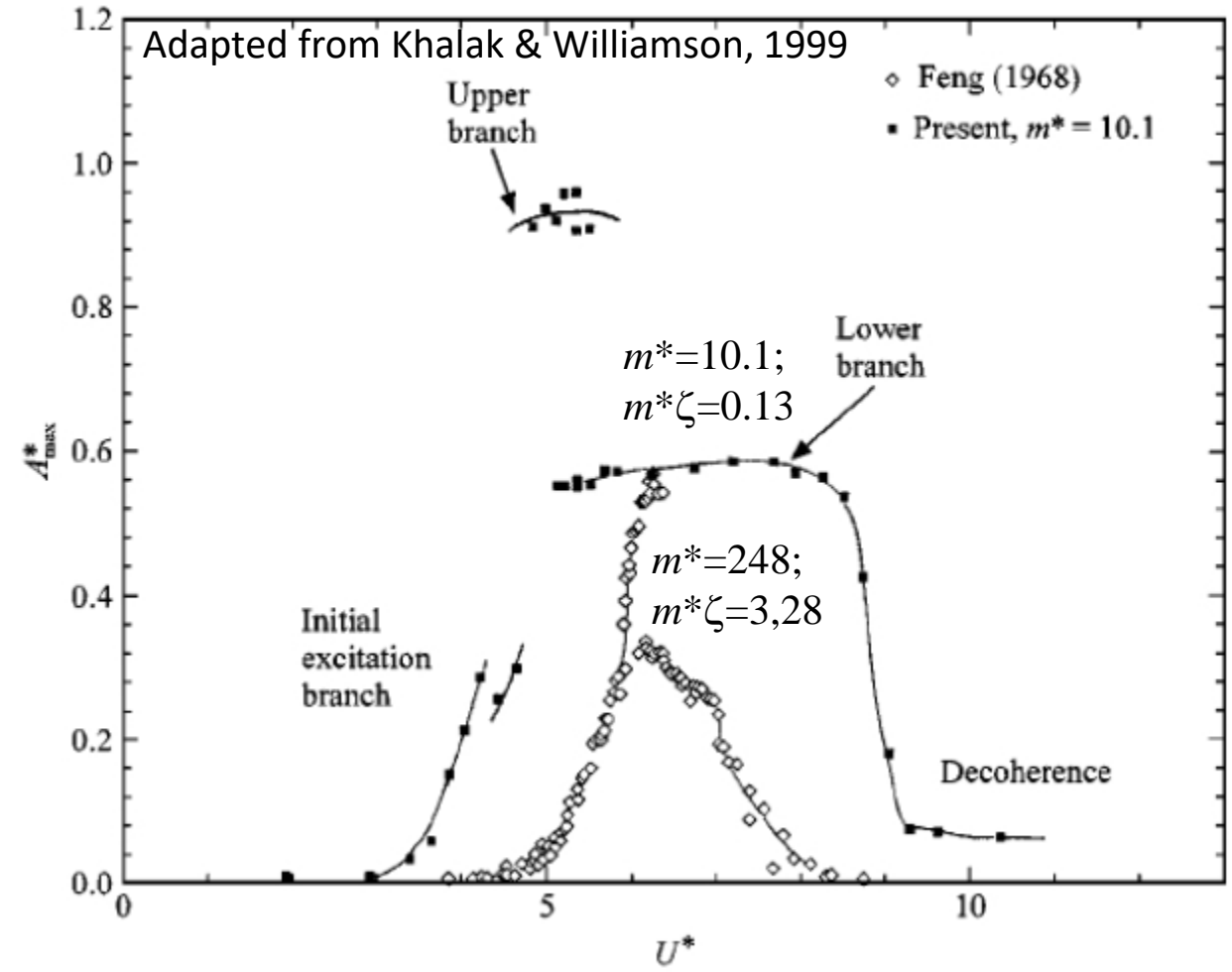
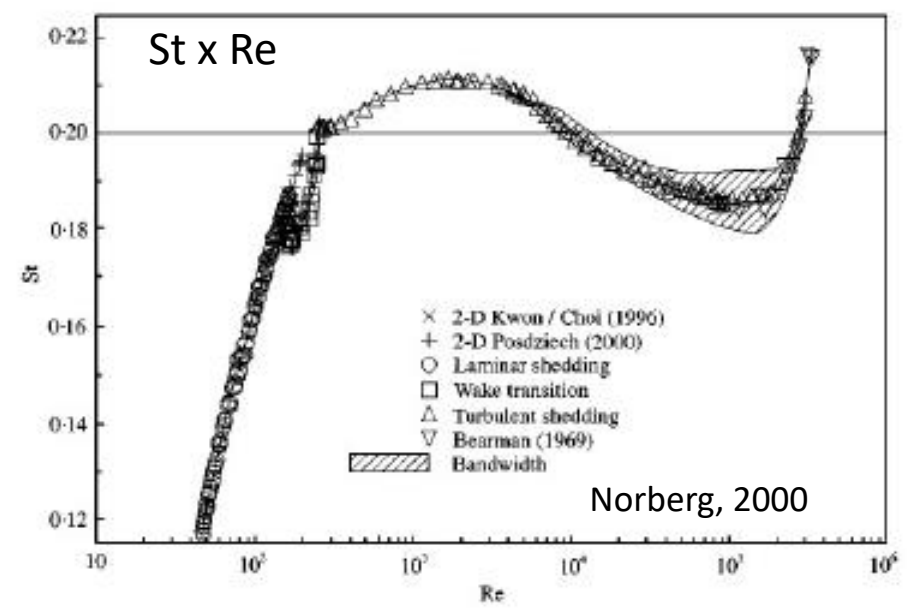
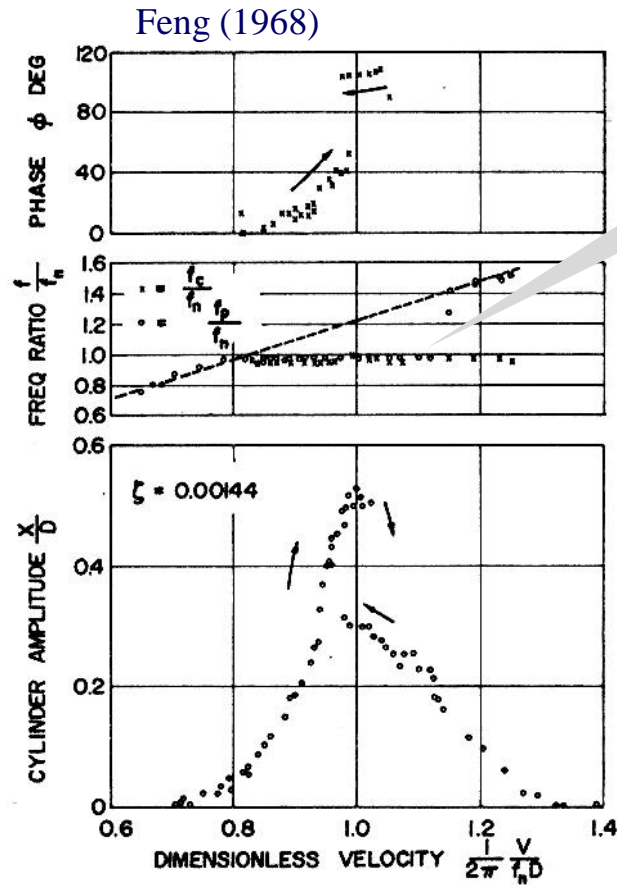


Figura 35: Variação do campo de pressão na parede para aproximadamente um terço do ciclo de emissão de vórtices. Adaptado de Blevins (1990) e Meneghini (1993).

# CLASSICAL VIV



# CLASSICAL VIV



Lock-in

Lock-in:

$$f_s^* = f_s / f_n = StU^* \cong 1$$

and

$$f^* = f / f_n \cong 1$$

if  $St \ll 0.2$  resonance peaks at  $U^* \cong 5$

$$\frac{y_0}{D} \propto \frac{C_Y \sin \phi}{(m^* + C_a) \zeta}$$

# Wake-oscillator model for 1DOF VIV considering Added Mass as function of reduced velocity and a stall term

Fujarra, 2002 (PhD thesis) and Fujarra & Pesce (ASME-FSI, 2002)

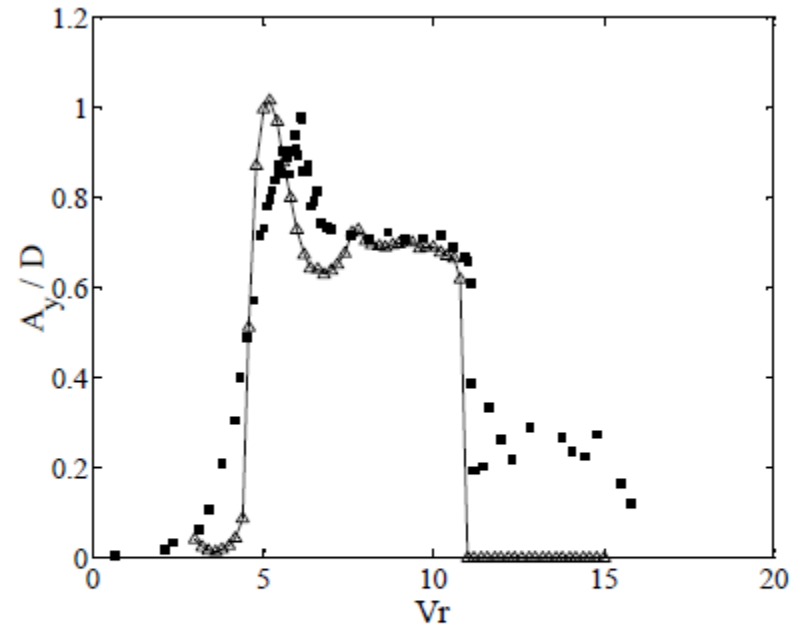
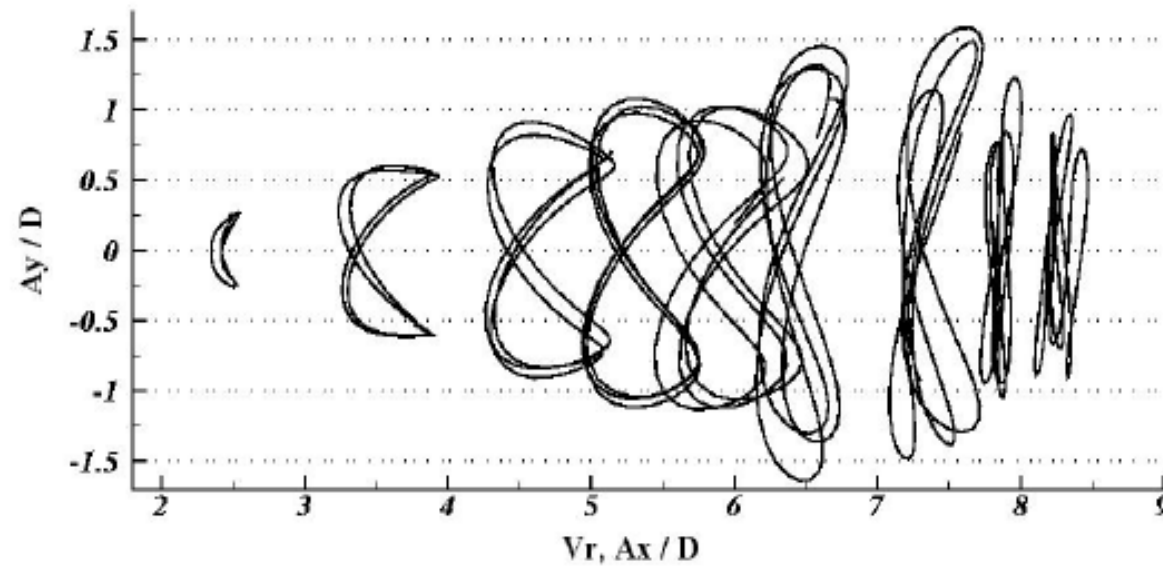


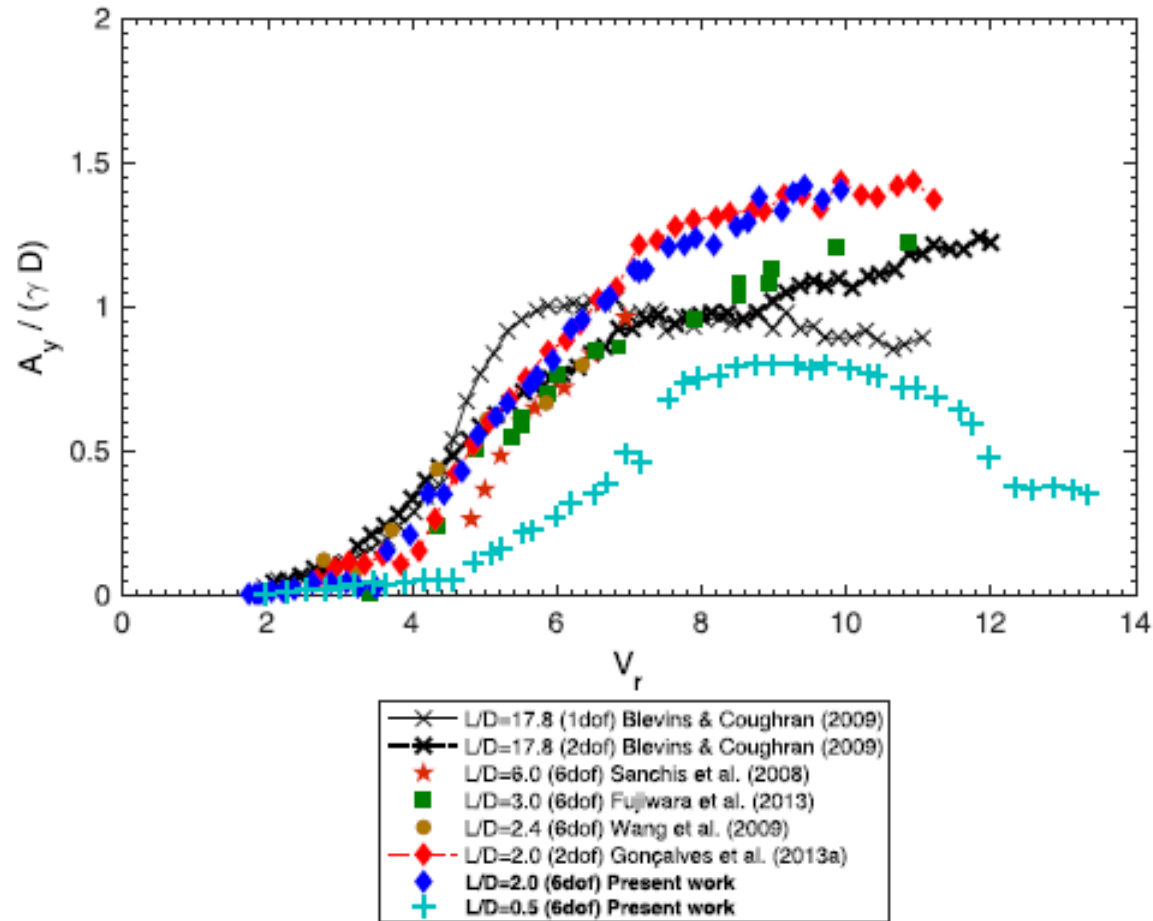
Figure 11. Non-dimensional amplitude prediction for Experiment (I) (Flexible Cantilever). Analytical Model modified by introducing added mass variability with reduced velocity, extracted from Experiment (II) (Elastically Mounted Rigid Cantilever, Figure 4). Lift-coefficient variability  $C_L = C_L(V_r)$ , adjusted according to Figure 10, after Willden and Graham (2001).

## 2DOF VIV



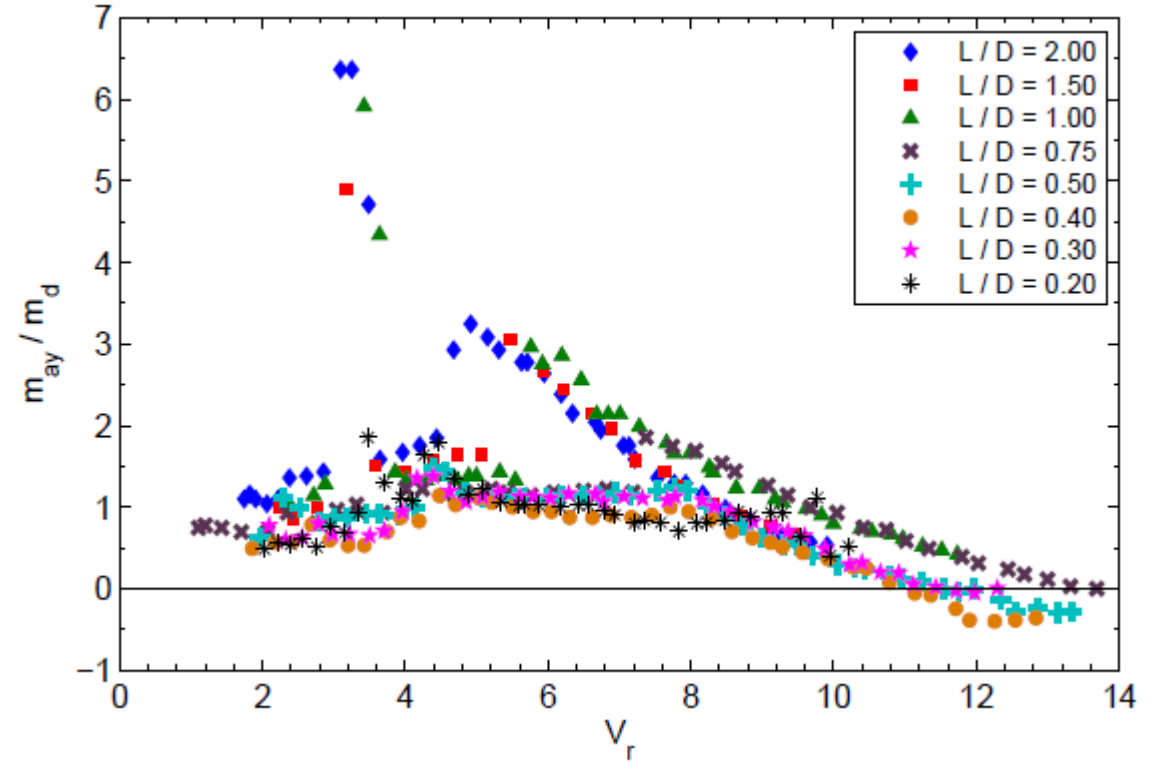
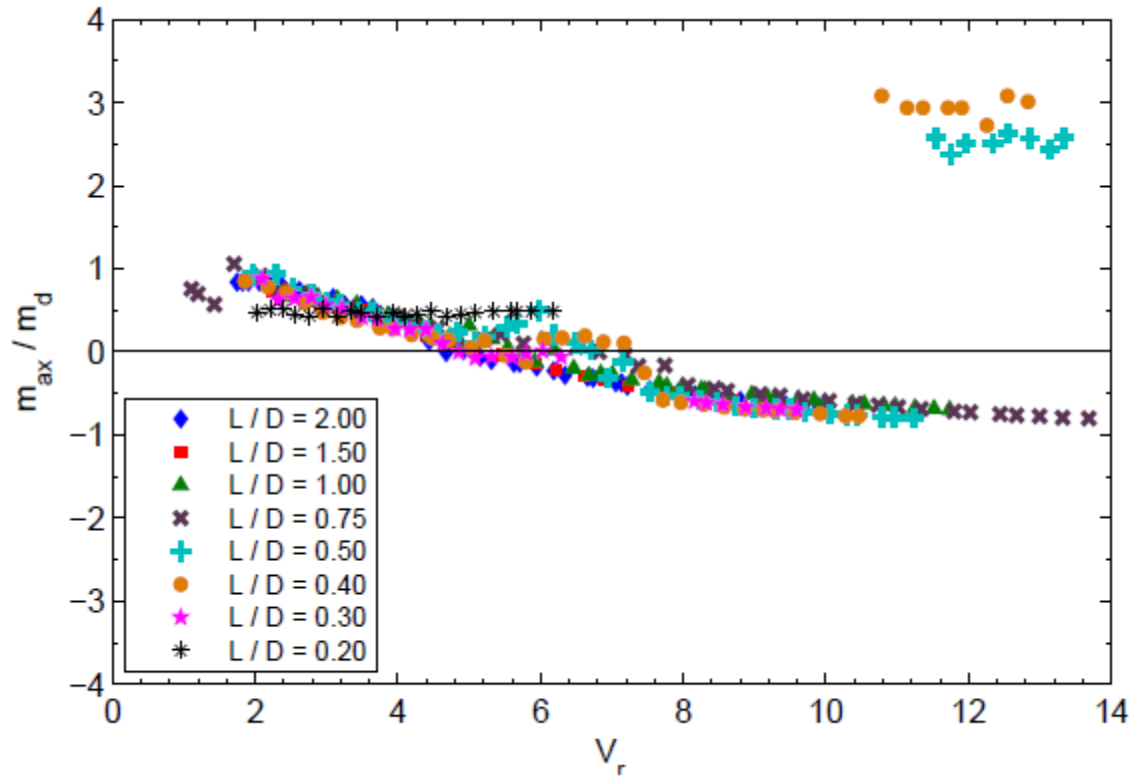
Eight-shaped trajectories: dual resonance

# CURRENT INDUCED MOTIONS ON LOW ASPECT RATIO CYLINDERS



Gonçalves et al., Ocean Engineering 2018

# CURRENT INDUCED MOTIONS ON LOW ASPECT RATIO CYLINDERS



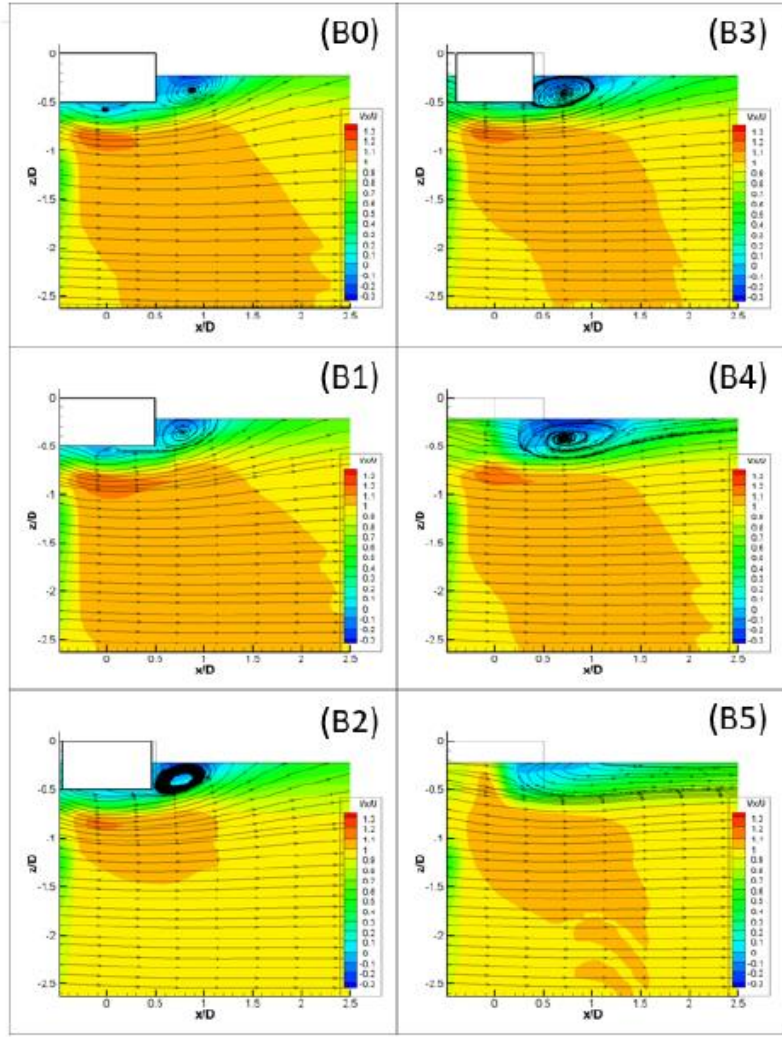
Added Mass in VIV  
Gonçalves, R.T., PhD Thesis, 2013



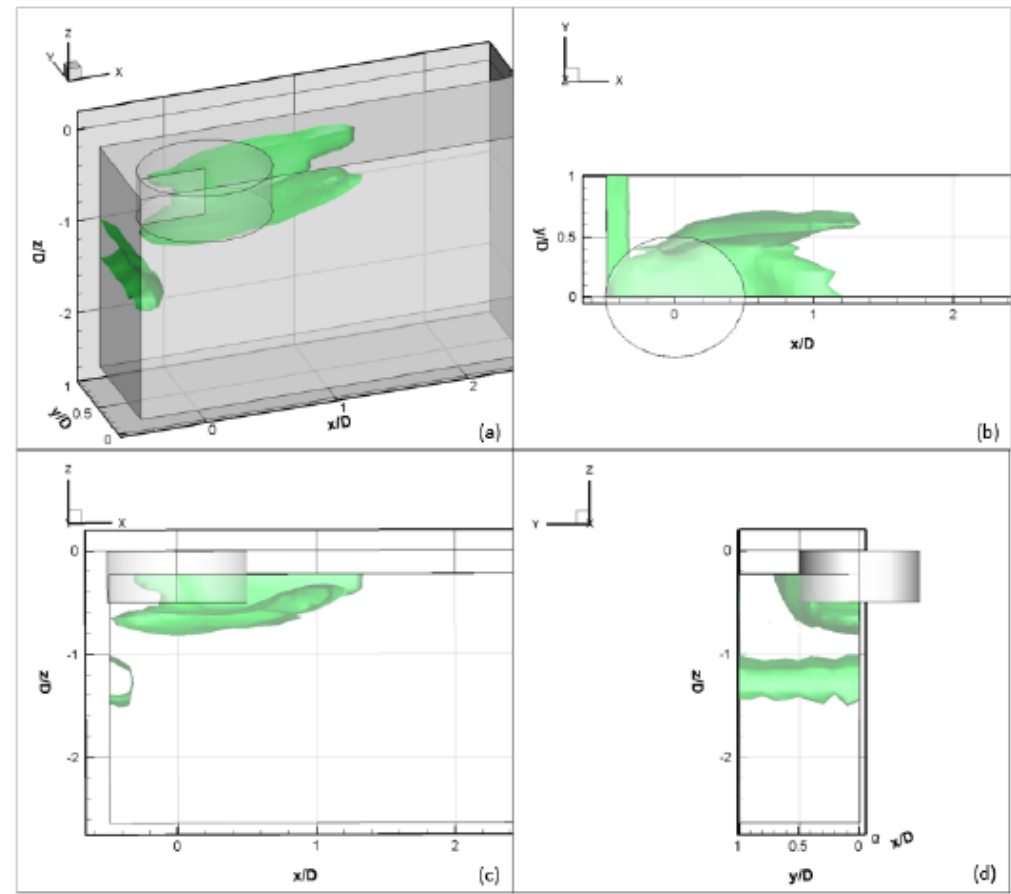
# FLOW AROUND A LOW-ASPECT RATIO CYLINDER (L/D=0.5)



SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS



$V_x/U$ ,  $Re=43000$



Vorticity isosurface  
 $Re=43000$

Gonçalves et al, 2018

# CURRENT INDUCED FORCES

(Inspired in Ogink and Metrikine, 2010  
and Franzini & Bunzel, 2018)

$$F_{Vx} = \frac{1}{2} \rho DLC_x V^2; \quad F_{Vy} = \frac{1}{2} \rho DLC_y V^2$$

$$C_x = (C_D U_x - C_L U_y) \frac{U}{V^2}; \quad C_y = (C_D U_y + C_L U_x) \frac{U}{V^2}$$

$$U_x = V - \dot{x}; \quad U_y = -\dot{y}; \quad U = \sqrt{U_x^2 + U_y^2}$$

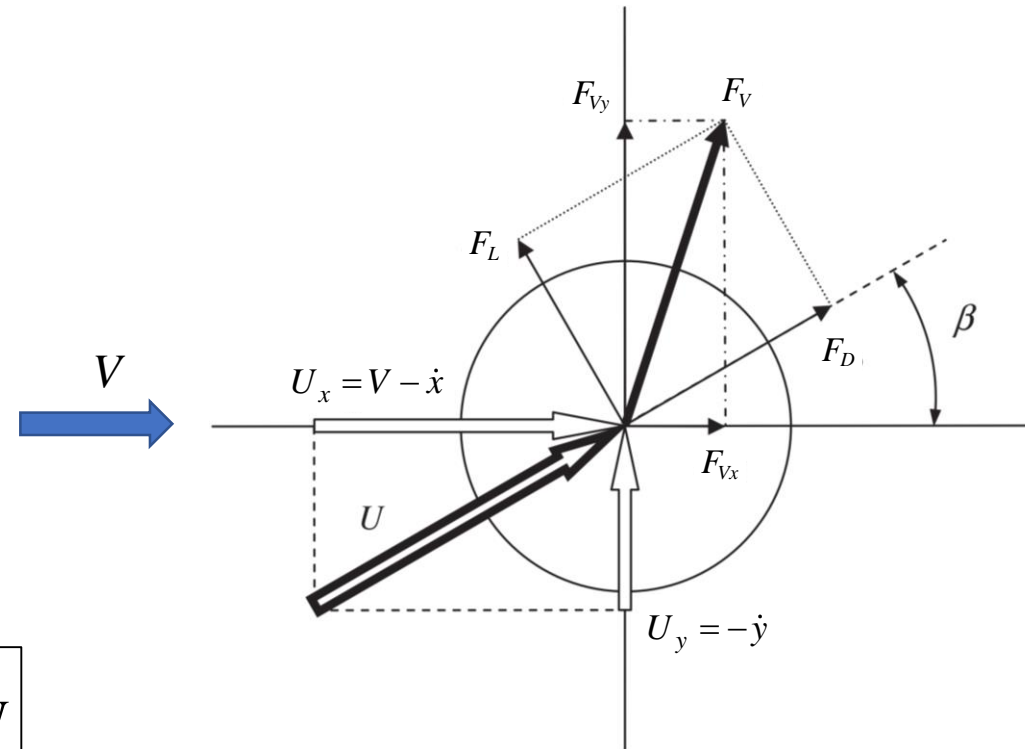
$$F_{Vx} = F_D \cos \beta - F_L \sin \beta; \quad F_{Vy} = F_D \sin \beta + F_L \cos \beta$$

$$\cos \beta = U_x / U; \quad \sin \beta = U_y / U$$

$$F_{Vx} = \frac{1}{2} \rho DH (C_D U_x - C_L U_y) U; \quad F_{Vy} = \frac{1}{2} \rho DH (C_D U_y + C_L U_x) U$$



$$\mathbf{Q}^v = \frac{1}{2} \rho DH U \left[ (C_D U_x - C_L U_y) \quad (C_D U_x - C_L U_y) \quad 0 \right]^T$$



# PHENOMENOLOGICAL MODEL AND HYDRODYNAMIC FORCES

**2 dof wake-oscillators of the van der Pol type:**

$$\ddot{w}_x + \varepsilon_x \omega_s (w_x^2 - 1) \dot{w}_x + 4\omega_s^2 w_x = \frac{A_x}{D} \ddot{x}$$

Enforced dual  
resonance

Inertial  
coupling

Shedding  
frequency

$$\ddot{w}_y + \varepsilon_y \omega_s (w_y^2 - 1) \dot{w}_y + \omega_s^2 w_y = \frac{A_y}{D} \ddot{y}$$

$$\omega_s = 2\pi S_t (V / D)$$

**governing two new generalized coordinates, related to the *wake dynamics*:**

$$\mathbf{w} = [w_x \quad w_y]^T$$

$(\varepsilon_x, \varepsilon_y)$  and  $(A_x, A_y)$

**Empirical parameters to be adjusted**

# PHENOMENOLOGICAL MODEL AND HYDRODYNAMIC FORCES



$$C_L = \frac{C_{L0}}{2} w_y$$

$$C_D = C_{D0} (1 + K w_y^2) + \frac{C_{D0}^f}{2} w_x$$

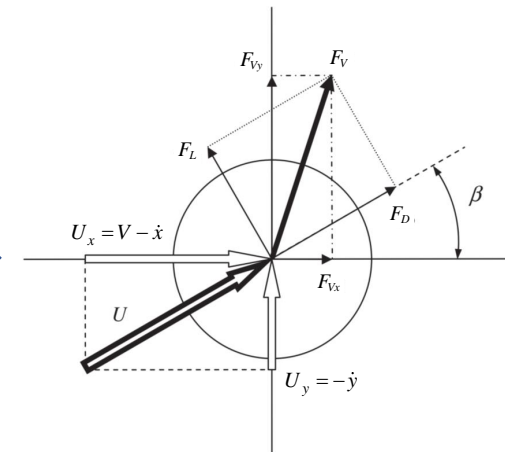


$$\mathbf{Q}^v = \frac{1}{2} \rho D H U \begin{bmatrix} (C_D U_x - C_L U_y) & (C_D U_x - C_L U_y) & 0 \end{bmatrix}^T$$

$C_{L0}$  and  $C_{D0}$  are the lift and drag coefficients for a fixed cylinder



$C_{D0}^f$  is a weighting coefficient for the oscillation amplitude of the drag  
 $K$  is an experimental constant



(Rosetti et al, 2009)



# THE 5-DOF REDUCED ORDER MODEL

$$\tilde{\mathbf{M}}\ddot{\tilde{\mathbf{q}}} = \tilde{\mathbf{Q}}_c + \tilde{\mathbf{Q}}_{nc}$$

$$\tilde{\mathbf{M}} \in \mathfrak{R}^{5 \times 5}; \tilde{\mathbf{q}} \in \mathfrak{R}^5$$

$$\tilde{\mathbf{M}} = \begin{bmatrix} M + M_a & 0 & 0 & 0 & 0 \\ 0 & M + M_a & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ -\frac{A_x}{D} & 0 & 0 & 1 & 0 \\ -\frac{A_y}{D} & 0 & 0 & 0 & 1 \end{bmatrix}; \tilde{\mathbf{q}} = \begin{bmatrix} \mathbf{q} \\ \mathbf{w} \end{bmatrix} = \begin{bmatrix} x \\ y \\ \psi \\ w_x \\ w_y \end{bmatrix}; \tilde{\mathbf{Q}}_c = \begin{bmatrix} \mathbf{Q}^m \\ -4\omega_s^2 w_x \\ -\omega_s^2 w_y \end{bmatrix}; \tilde{\mathbf{Q}}_{nc} = \begin{bmatrix} \mathbf{Q}^v \\ -\varepsilon_x \omega_s (w_x^2 - 1) \dot{w}_x \\ -\varepsilon_y \omega_s (w_y^2 - 1) \dot{w}_y \end{bmatrix}$$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

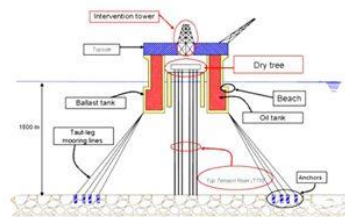


## Monocoluna - Moonpool

- MonoBR



▫ Completação Seca



**Table 1 – MonoBr-GoM main particulars and general parameters.**

Draught, $H$ (m)	39.50
Diameter, $D$ (m)	100.0
Mass, $M$ (t)	262000.0
Added mass coefficient, $C_a$	1.0
Mass of water in moon-pool, $M_{mp}$ (t)	67447.0
Density of water, $\rho$ (t/m <sup>3</sup> )	1.025

**Table 3 – Wake-oscillators parameters; Rosetti et al (2009), Gonçalves et al (2010).**

$[A_x; A_y]$	[12; 6]
$[\varepsilon_x; \varepsilon_y]$	[0.30; 0.15]
$[C_{D0}; C_{L0}; C_{D0}^f; K]$	[0.70; 0.30; 0.10; 0.05]
Strouhal number, $S_f$	0.078



# MONOCOLUMN MOORING SYSTEM

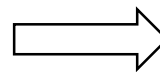
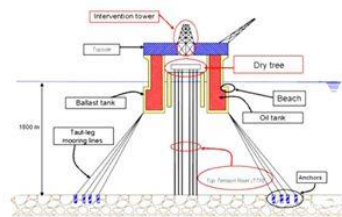
Plataformas de Petróleo e Gás

## Monocoluna - Moonpool

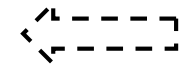
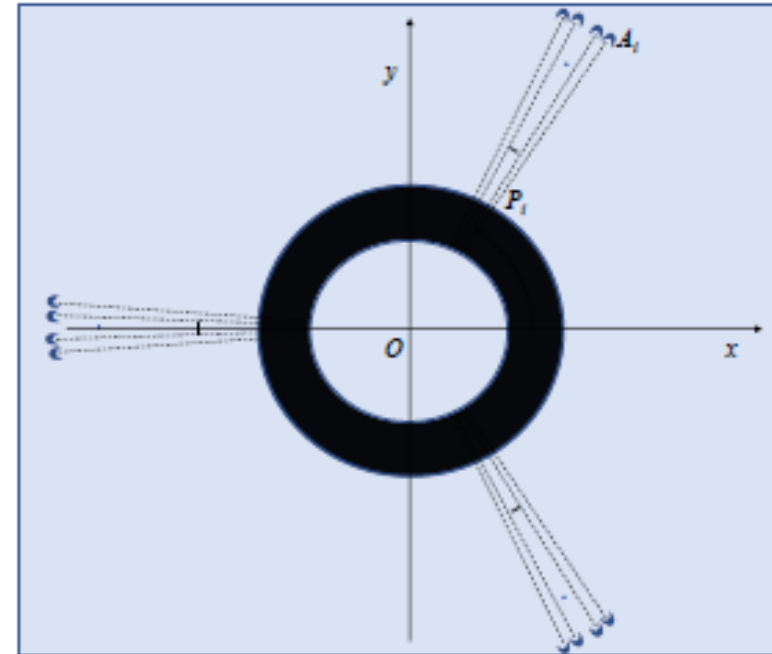
- MonoBR



□ Completação Seca



$$\alpha = 0$$

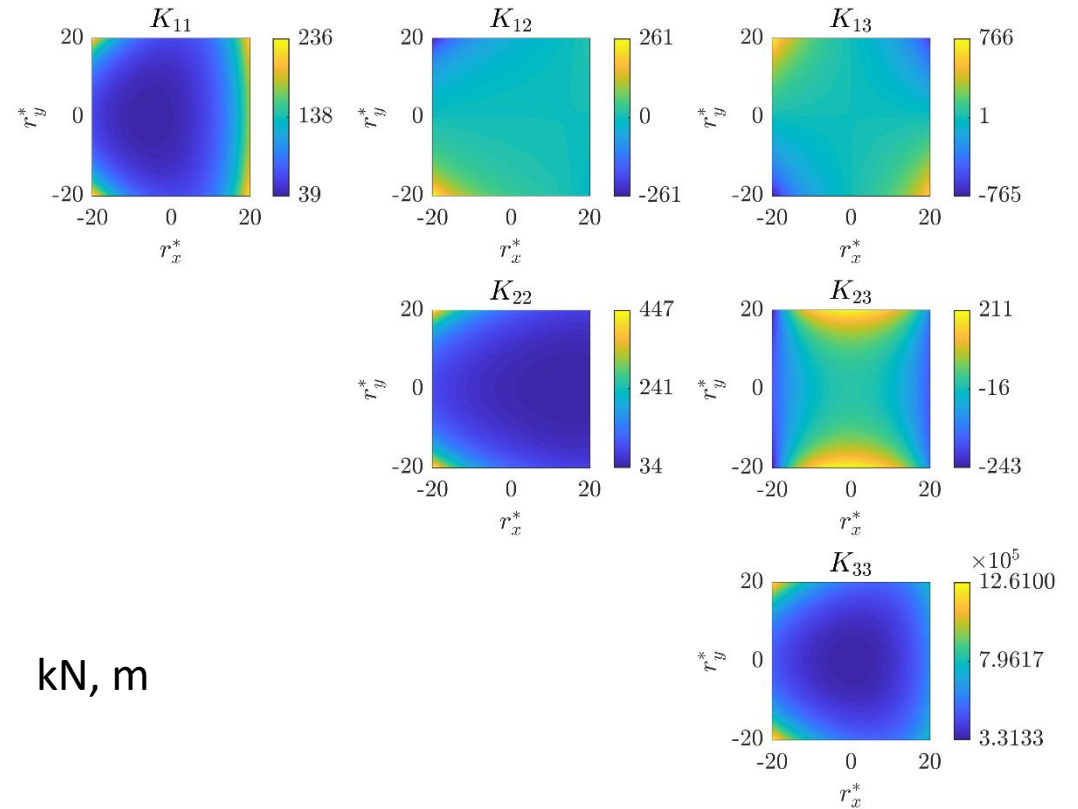
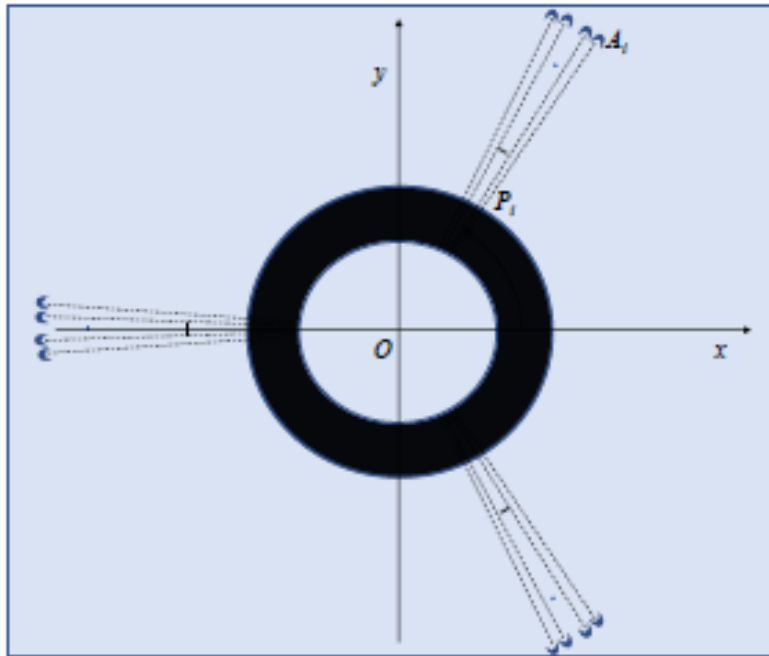


$$\alpha = \pi$$

Table 2 – Mooring system: parameters and arrangement at neutral position,  $q = 0$ . **Catenary chains. Depth 500m.**

Immersed weight, $\gamma$ (kN/m)	1.07
Total length, $L$ (m)	1525.0
Distance of fairlead from anchor, $r(f_0)$ (m)	1317.0
Horizontal component of pretension, $f_0$ (kN)	532.83
Number of mooring lines, $N$	12
Angles of mooring lines in the body frame, $\beta$ (rad)	[(0.79, 0.96, 1.13, 1.31); (2.88, 3.05, 3.22, 3.40); (-0.79, -0.96, -1.13; -1.31)]

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)



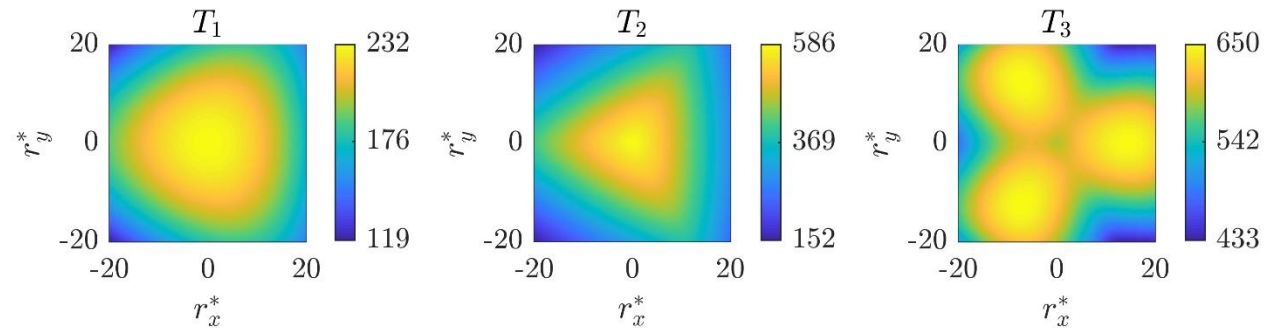
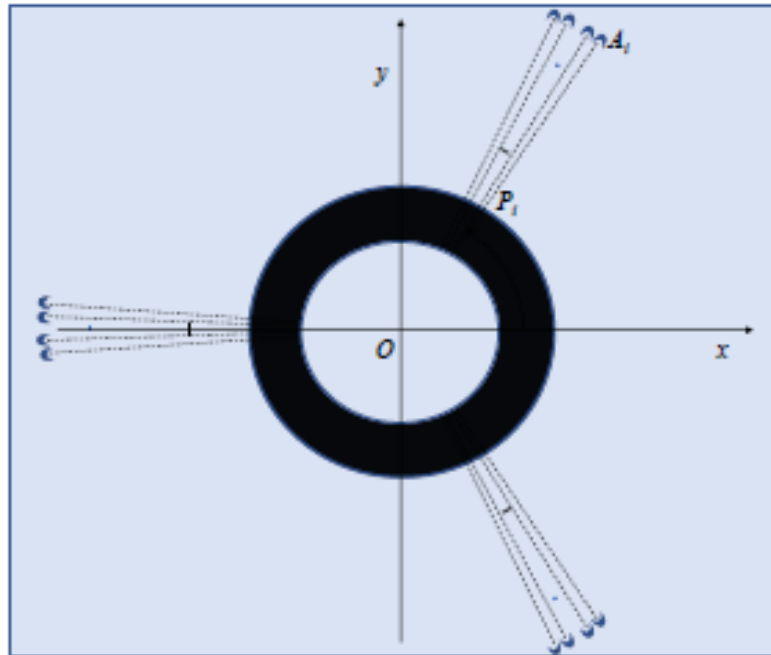
kN, m

Local mooring stiffness matrix as function of platform offset (% depth)  
at yaw angle  $\Psi = 0^\circ$



# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

Perfectly symmetric mooring arrangement

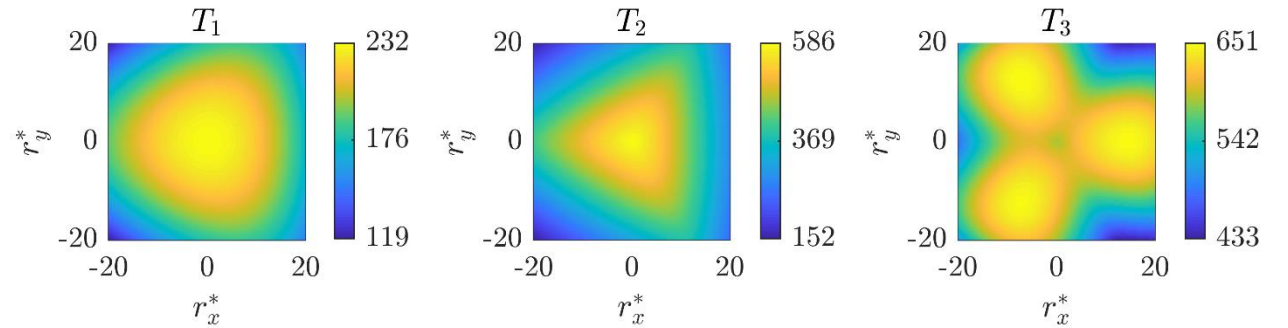
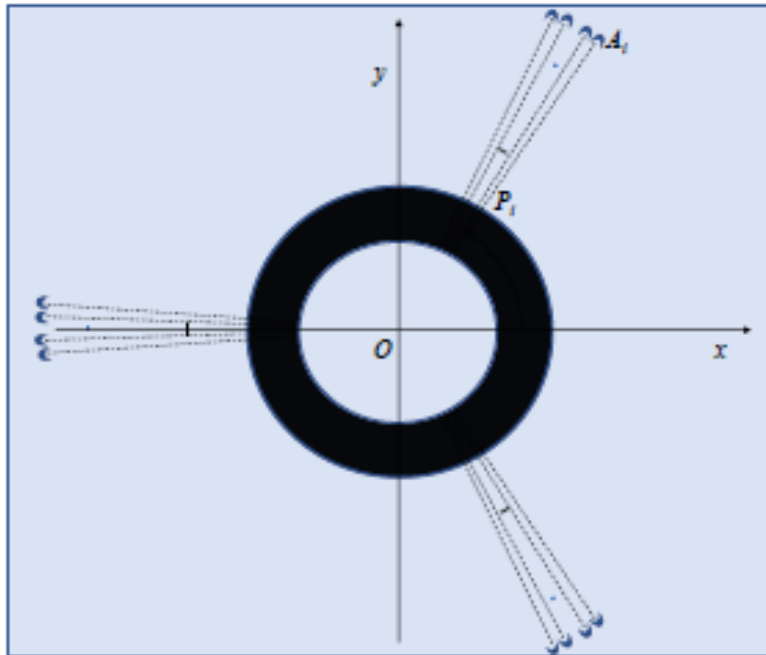


Natural periods as function of platform offset (% depth)  
at yaw angle  $\Psi = 0^\circ$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

Small changes in symmetric mooring arrangement

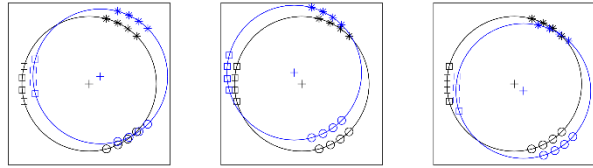
179.76 °



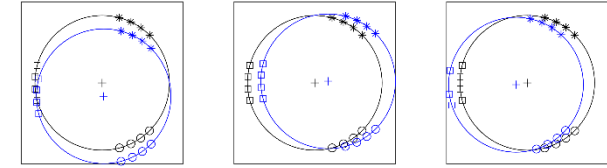
Natural periods as function of platform offset (% depth)  
at yaw angle  $\Psi = 0^\circ$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

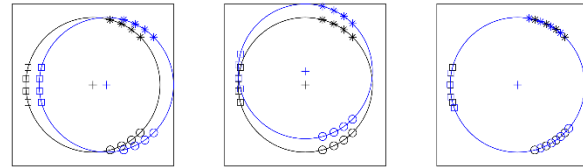
Perfectly symmetric mooring arrangement



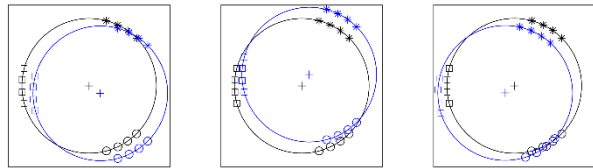
$(-100, 100)$



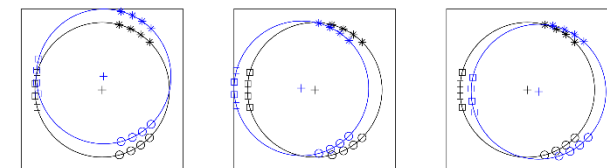
$(100, 100)$



$(0, 0)$



$(-100, -100)$

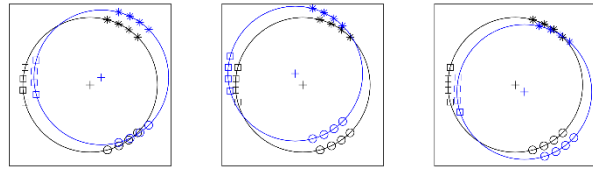


$(100, -100)$

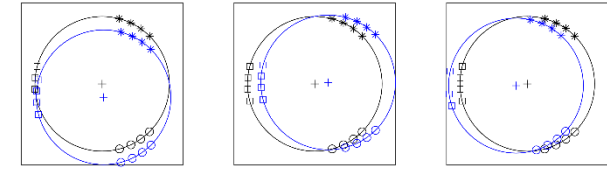
Oscillation modes for five offsets (m).

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

Small changes in symmetric mooring arrangement

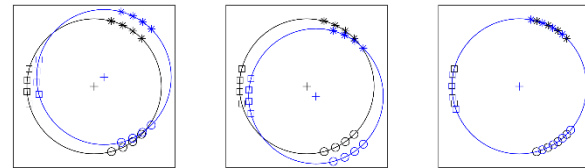


(-100,100)

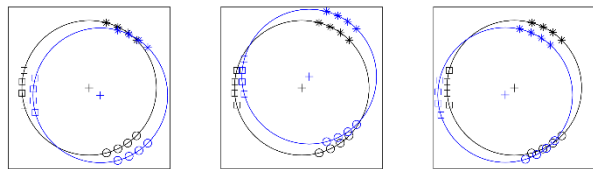


(100,100)

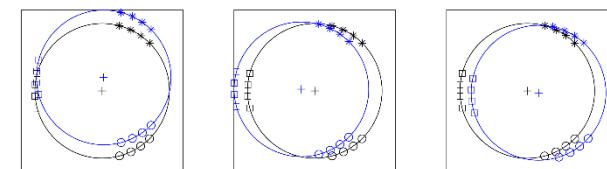
179.76 °



(0,0)



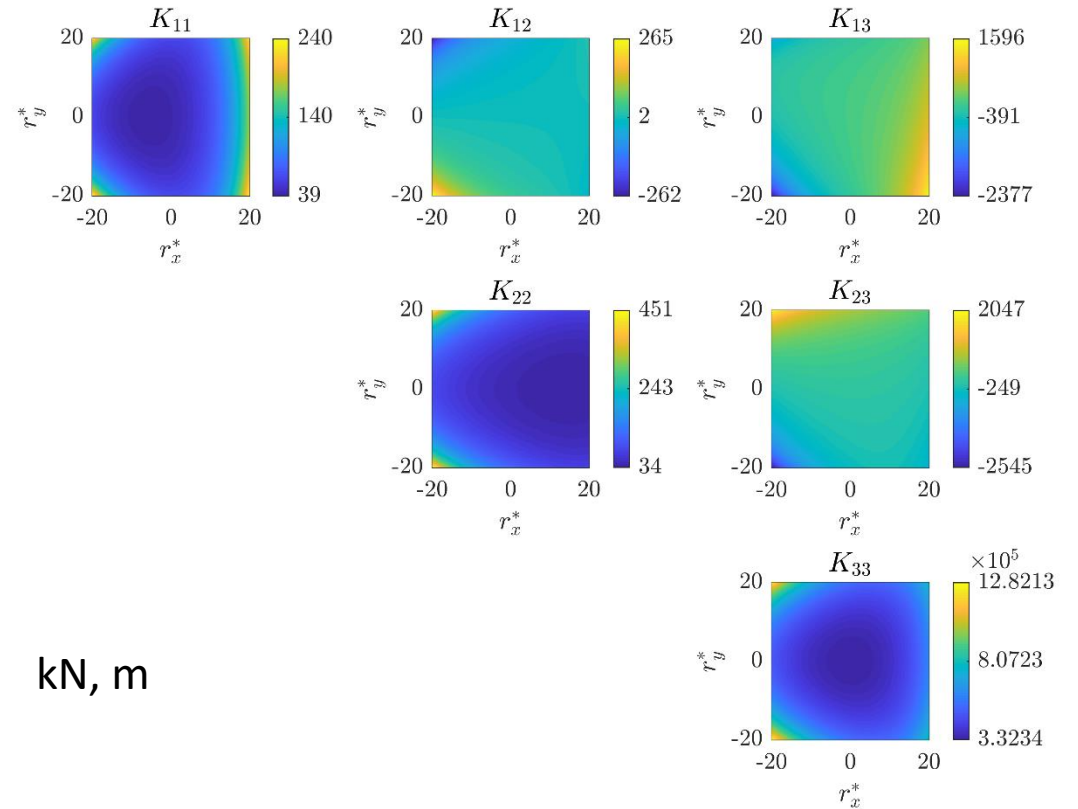
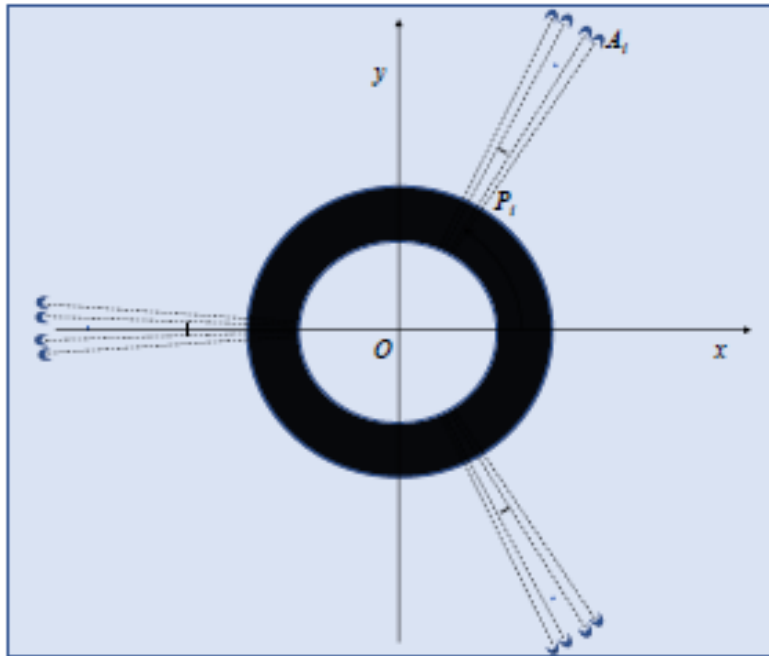
(-100,-100)



(100,-100)

Oscillation modes for five offsets (m).

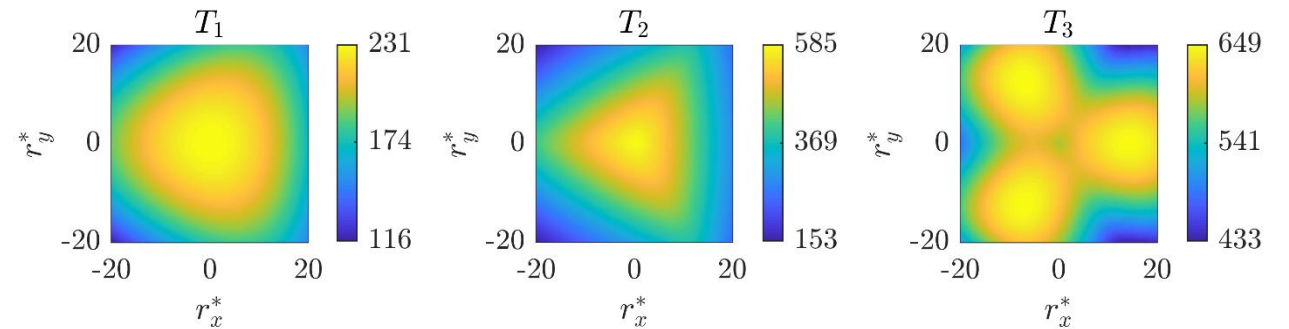
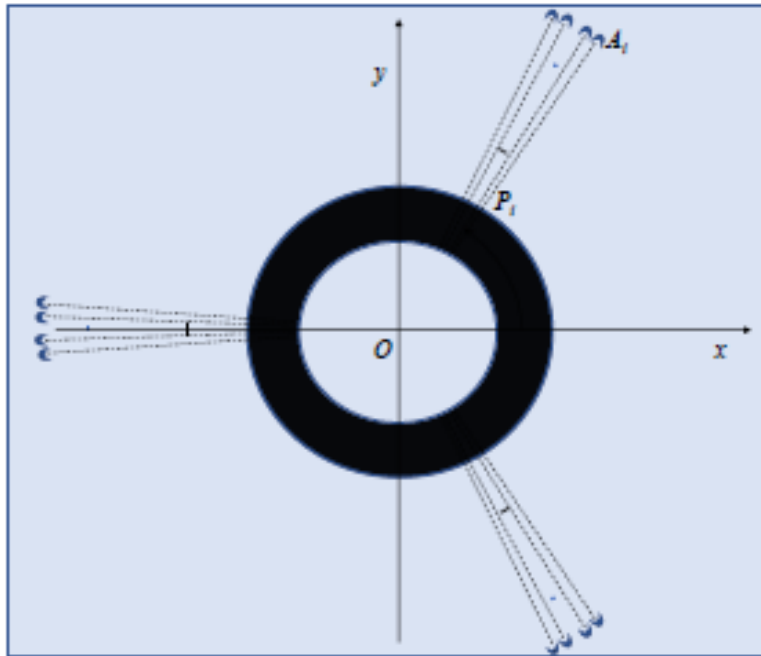
# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)



Local mooring stiffness matrix as function of platform offset (% depth)  
 at yaw angle  $\Psi = 5^\circ$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)


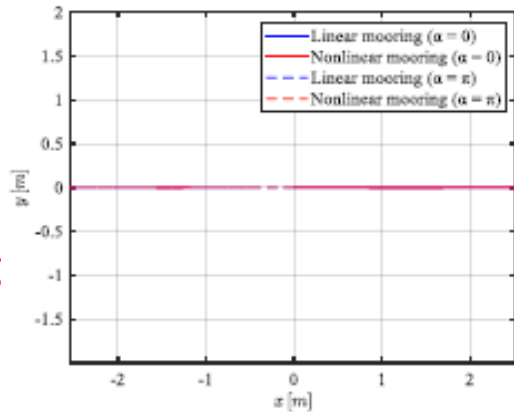
Small changes in symmetric mooring arrangement



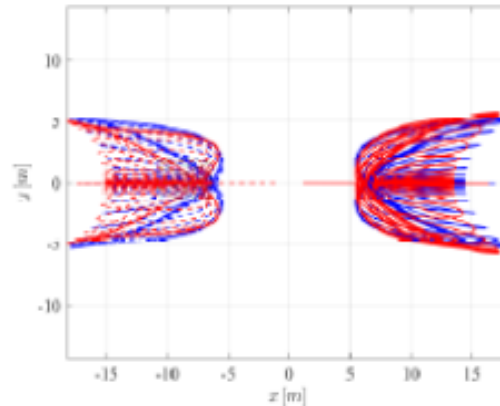
Natural periods as function of platform offset (% depth)  
at yaw angle  $\Psi = 0^\circ$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

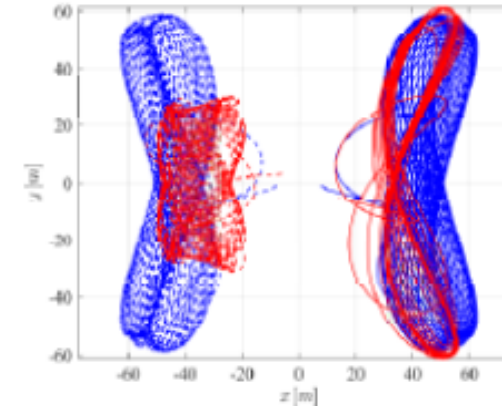
Nonlinear mooring  
 $\alpha = 0$

(a):  $V = 0,20$  (m/s);  $V_r = 1,57$ .




(b):  $V = 0,56$  (m/s);  $V_r = 4,40$ .


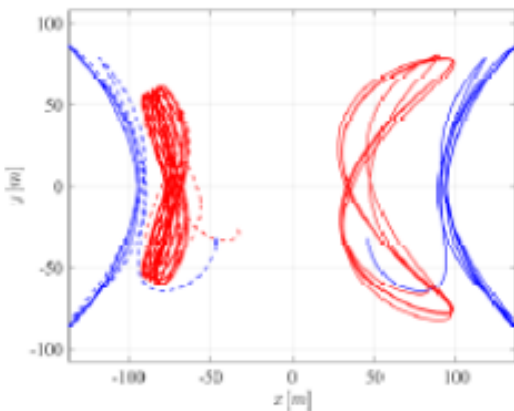


(c):  $V = 0,92$  (m/s);  $V_r = 7,23$ .

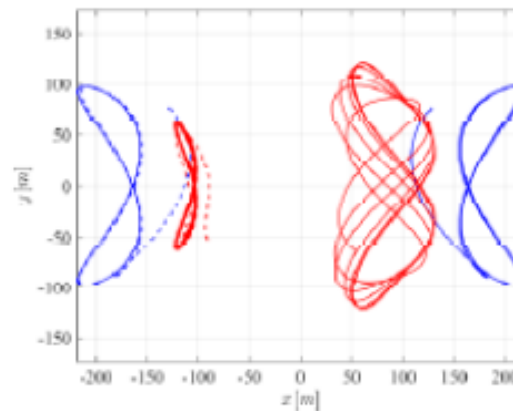
Nonlinear mooring  
 $\alpha = \pi$



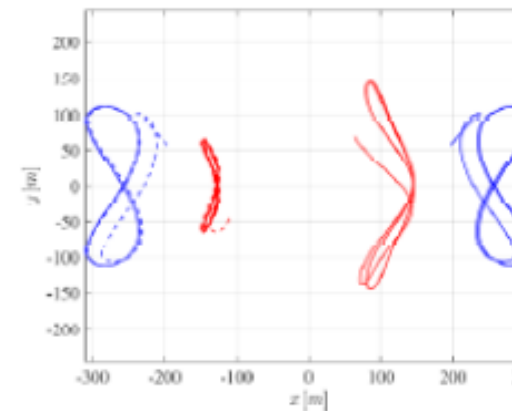
Linear mooring  
 $\alpha = 0$

(d):  $V = 1,28$  (m/s);  $V_r = 10,05$ .



(e):  $V = 1,64$  (m/s);  $V_r = 12,88$ .



(f):  $V = 2,00$  (m/s);  $V_r = 15,71$ .

Linear mooring  
 $\alpha = \pi$


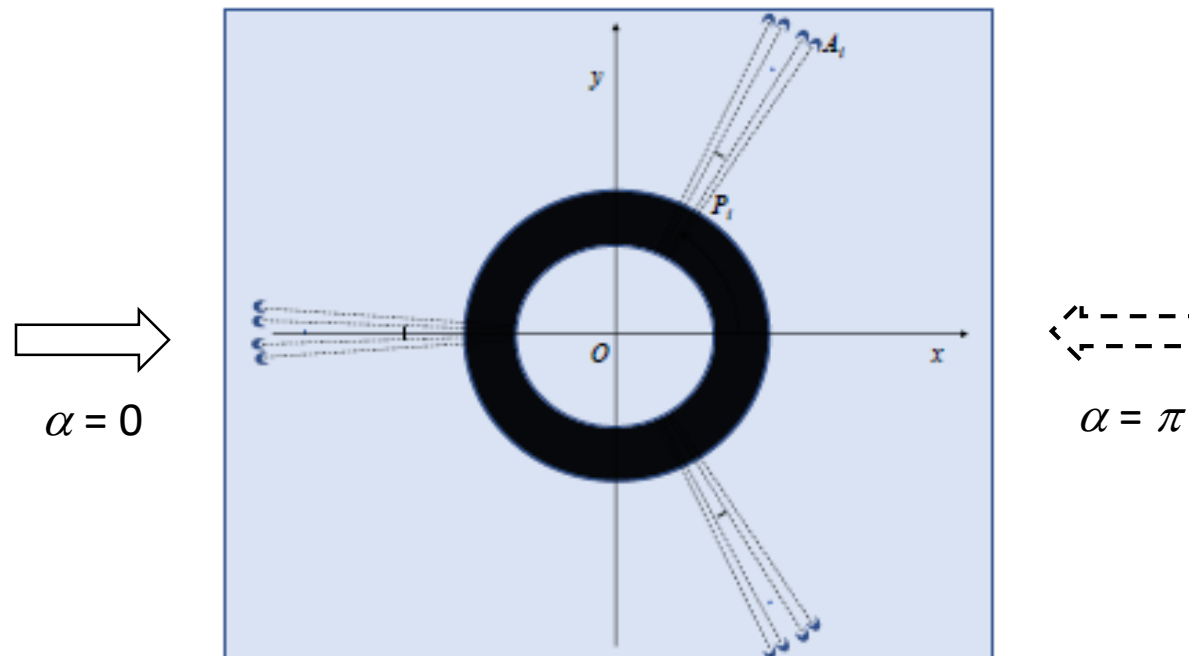


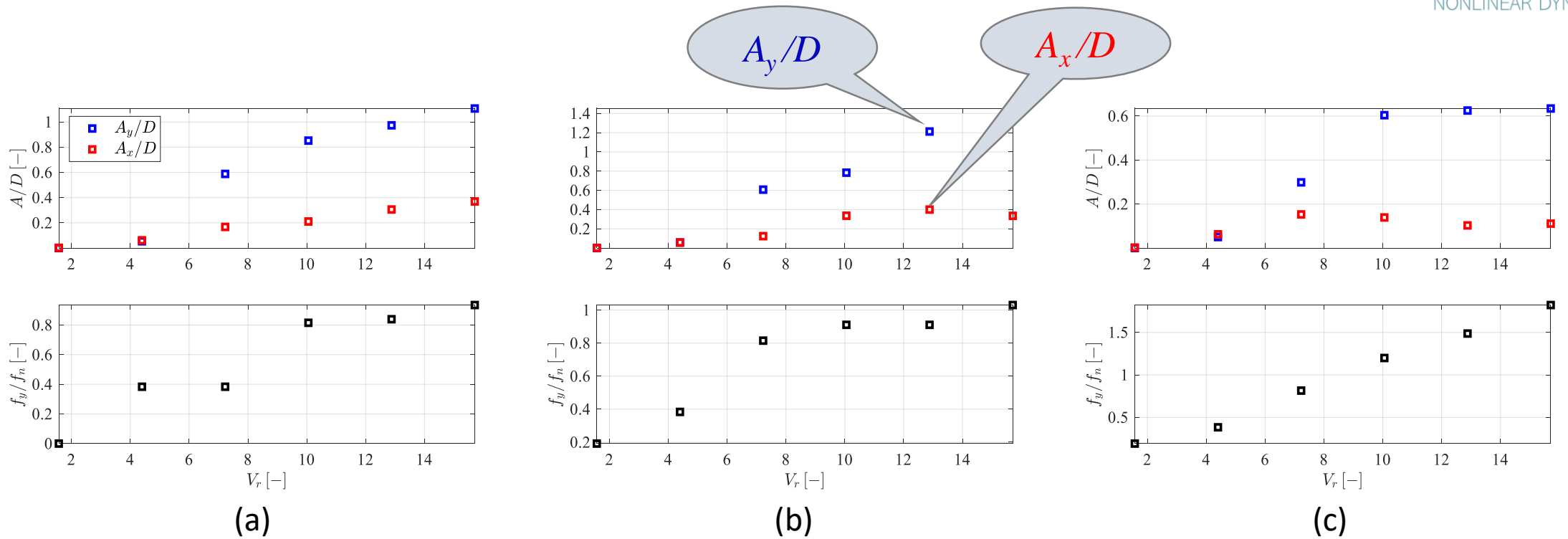
Figure 6 – Center of mass trajectories on the horizontal plane. Current direction: (i) from left to right,  $\alpha = 0$  and (ii) from right to left,  $\alpha = \pi$ . The terms ‘linear’ and ‘nonlinear’ in the legend refer only to the mooring system.

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)





# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)



**Figure 7 – Normalized amplitudes and oscillation frequencies, as a function of the reduced velocity. (a): linear mooring model,  $\alpha = 0, \pi$ ; (b): nonlinear mooring model,  $\alpha = 0$ ; (c): nonlinear mooring model,  $\alpha = \pi$ .**

$St = 0.078 \Rightarrow$  resonance peak expected at  $V_r = 1/St \cong 12.8$

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

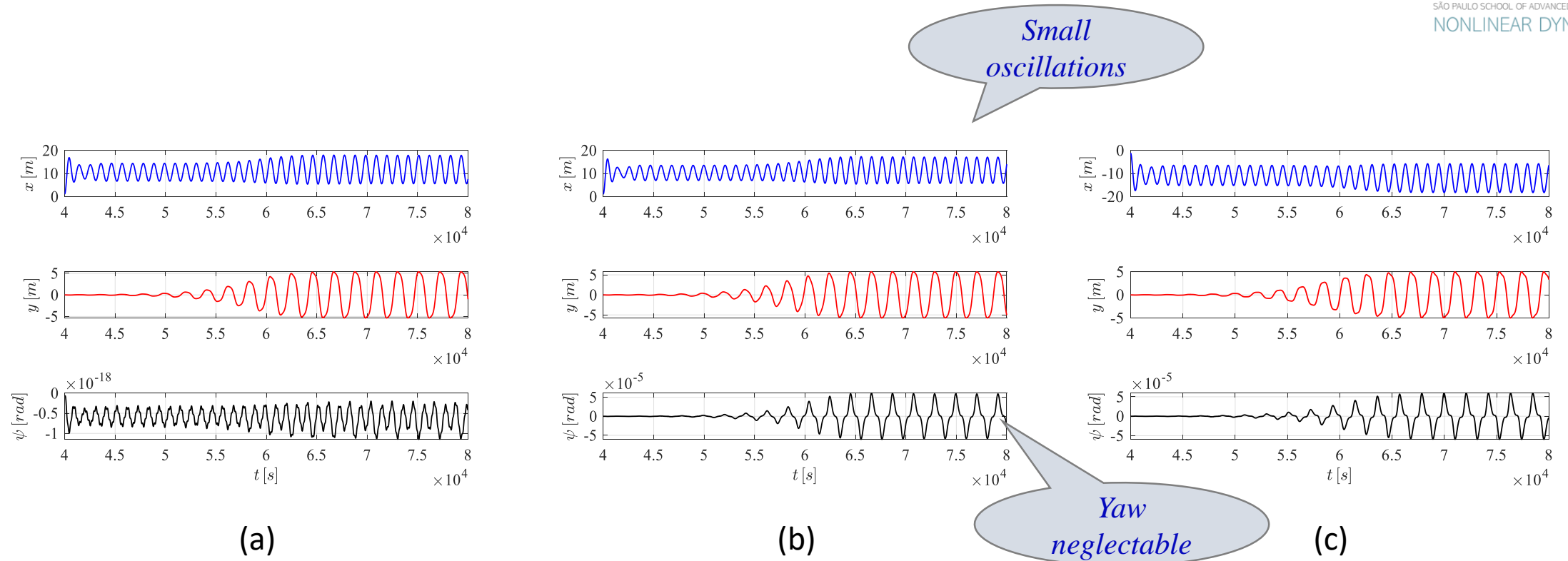


Figure 8. Surge, sway and yaw motions.  $V=0.56\text{m/s}$ ;  $V_r=4.4$ .

(a): linear mooring model,  $\alpha = 0, \pi$ , (b): nonlinear mooring model,  $\alpha = 0$ ; (c): nonlinear mooring model,  $\alpha = \pi$ .

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

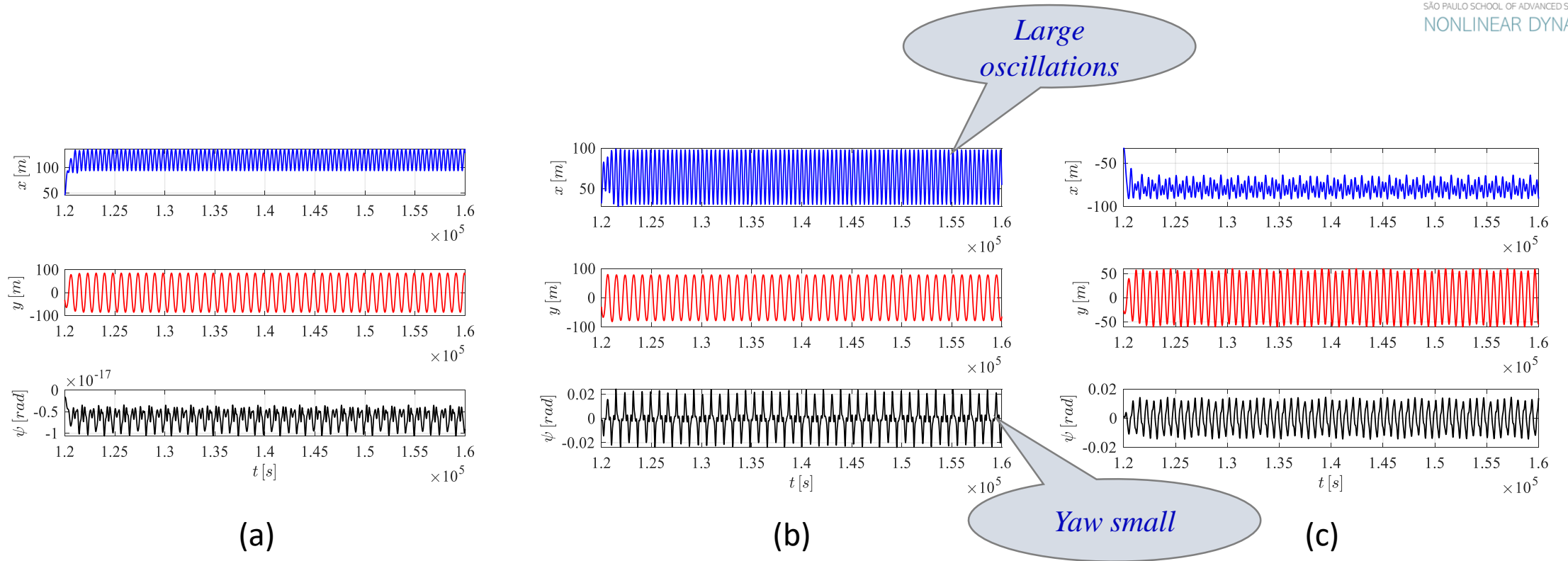


Figure 8. Surge, sway and yaw motions.  $V=1.28\text{m/s}$ ;  $V_r=10.0$ .

(a): linear mooring model,  $\alpha = 0, \pi$ , (b): nonlinear mooring model,  $\alpha = 0$ ; (c): nonlinear mooring model,  $\alpha = \pi$ .

# CASE STUDY - A MONOCOLUM PLATFORM (L/D=0.4)

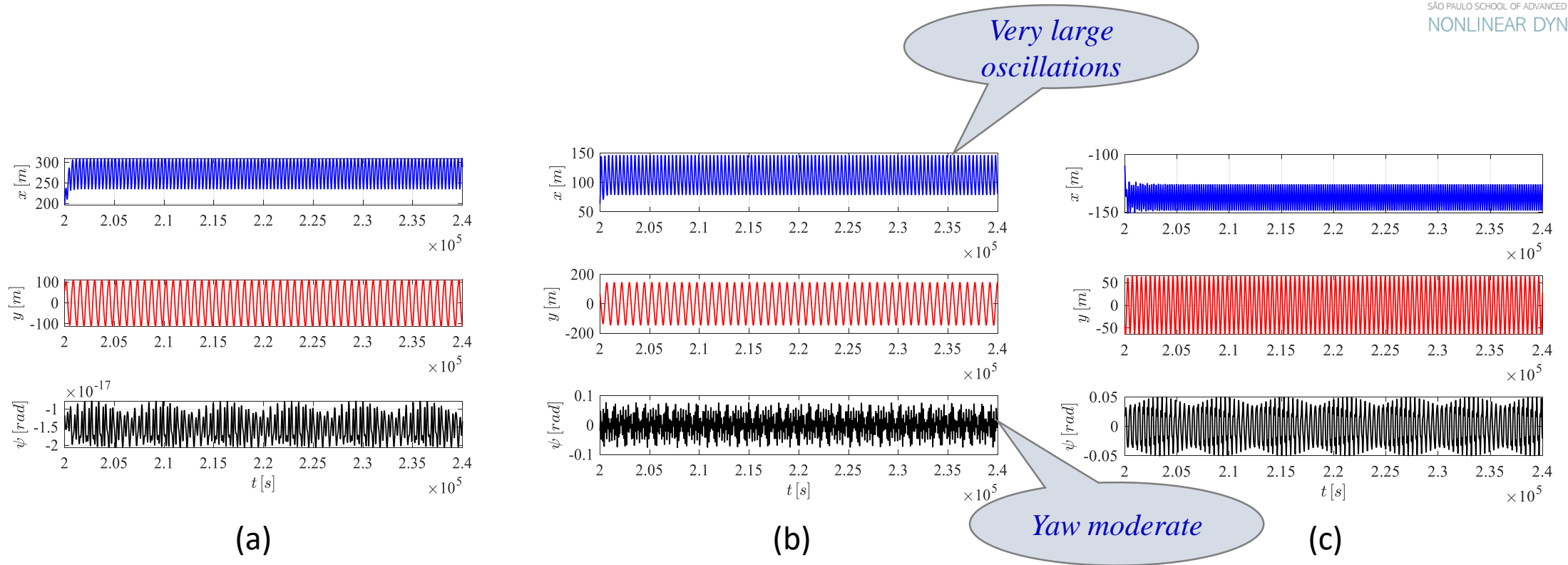
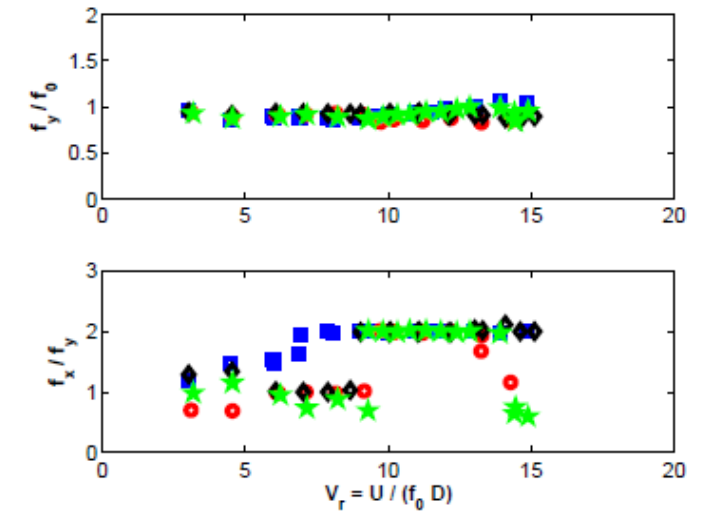
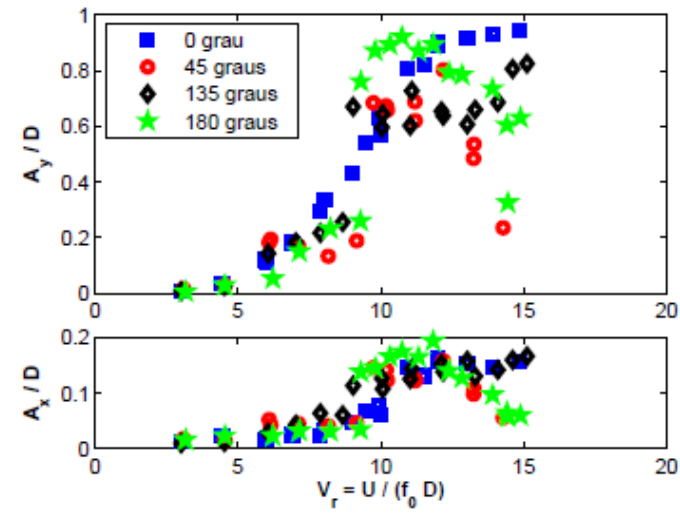
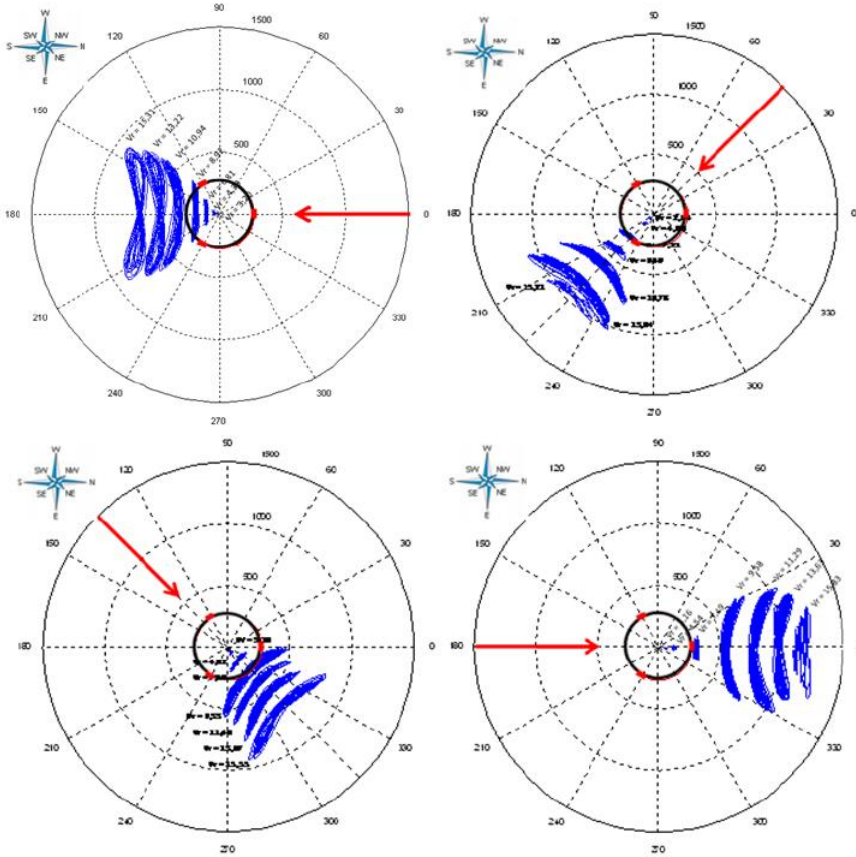


Figure 8. Surge, sway and yaw motions.  $V=2.0\text{m/s}$ ;  $V_r=15.7$ .

(a): linear mooring model,  $\alpha = 0, \pi$ , (b): nonlinear mooring model,  $\alpha = 0$ ; (c): nonlinear mooring model,  $\alpha = \pi$ .

# MONOBR – TRAJECTORIES AND DYNAMIC RESPONSE



MonoBR, Experimental data, *linear mooring lines*  
 Gonçalves, R.T., PhD Thesis, 2013  
 Gonçalves et al 2010, J Offshore Mech and Arctic Engineering

# ACKNOWLEDGMENTS

## ❖ LMO team, particularly:

- Dr. Guilherme R. Franzini, Associate Professor
- Dr. Renato M.M. Orsino, Assistant Professor
- Giovanni A. Amaral, PhD student
- Wagner A. Defensor Filho, PhD student
- Rafael Salles, PhD student
- Letícia S. Madi, MSc student
- Renato Finoteli, MSc student

## ❖ NDF team, particularly:

- Dr. Gustavo R.S. Assi, Assistant Professor
- Professor Julio R. Meneghini

## ❖ TPN team, particularly:

- Dr. Pedro C. de Mello, TPN Research Lab
- Professor Eduardo A. Tannuri
- Professor Kazuo Nishimoto

## ❖ Dept of Mechanical Engineering, particularly:

- Dr. Décio C. Donha, Associate Professor and Chair
- Everton L. de Oliveira, PhD student

## ❖ Dept of Ocean Engineering, particularly:

- Dr. Bernardo L. R. Andrade, Chair
- Dr. Alexandre N. Simos, Associate Professor

## ❖ Ocean Space Utilization Lab, University of Tokyo, particularly:

- Dr. Rodolfo T. Gonçalves, Assistant Professor

# SUPPORTING AGENCIES





SÃO PAULO SCHOOL OF ADVANCED SCIENCES ON  
NONLINEAR DYNAMICS



**ESCOLA POLITÉCNICA DA  
UNIVERSIDADE DE SÃO PAULO**

